



# Techie Math: Cool Uses of Technology in Higher Mathematics

Using the TI-83/84  
Graphing Calculator  
with higher math problems.



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# Introduction

Technology is a great way to teach new math concepts, create math interest in students who otherwise would not be interested, or reinforce already taught learned math concepts. I'm sure you've already figured out that, in most cases, it's we teachers that need to keep up with what our students already know with regard to today's technology. My five-year-old daughter knows how to navigate her way around certain kid-friendly web-sites better than I do, and my educational background is in computers and software. The invention and broader use of cell phones and MP3 players has certainly changed the way our students see the world around us, compared to the long-gone days of when we were in school.

Here's a chance for you to involve your students in high school level math using technology that they may not already be familiar with. With the use of a Texas Instruments (TI) graphing calculator, your already tech-savvy students, as well as those just learning the latest gadgets, can benefit from today's educational technology.

## ***Assumptions***

The following lessons in this math unit will make a couple of major assumptions. Let's be clear about them from the start.

- 1) I will assume you have access to at least one TI-83/84 graphing calculator. If not, see the resources section at the end of this unit for more information on how to purchase these calculators from reputable educational suppliers.
- 2) The math concepts in this unit have already been taught to students, or at least you are familiar enough with the concepts yourself that you can quickly teach them to your students. Although there is a little "teaching" happening in this unit, ***these lessons emphasize how to use the technology and not the teaching of the math concepts.***

## ***Technology***

The TI graphing calculator that will be used in the following lessons can either be the TI-83, TI-83 Plus, or the TI-84 Plus. Any of these calculators will be able to perform the algebraic and trigonometric operations shown throughout this unit. When a school kit or package of these calculators is purchased, the kit usually contains a poster-sized picture of the calculator. Hanging the poster up in your classroom in a location with easy access to it could help you point to specific buttons during demonstrations to



your students. Also, Texas Instruments has created software that emulates their graphing calculators, which can be shown using a computer connected to a video projector. This software is referenced at the end of the unit. It can also show students button-by-button steps which you took to achieve the results you attained.

Late edition math textbooks typically include a technology section that coincides with the math lesson(s) to be taught. Once you feel comfortable enough using the TI graphing calculator, try some of the activities or lessons that are found in your textbook. If the edition of text you use in your math class does not include such technology, you may request a complimentary desk copy from the major educational publishers to use as a reference.

## ***Buttons & Menus***

One more thing before we begin. In the first several lessons I will show you which buttons to push on your calculator.

Using the TI calculator, it will look like the following:

$\text{SIN} \rightarrow 37 \rightarrow \text{ENTER}$

This syntax means you are to push the “SIN” button located toward the middle of the calculator, followed by the number 37, followed by the “ENTER” button located in the lower right-hand corner of the calculator. Doing this will give you the sine of 37 degrees.

At times you will need to use the yellow-colored “2nd” button located in the upper left-hand corner of the calculator, before accessing the actual desired button. If the function you need to access is not written in white and located directly on top of a button, but is yellow in color and located slightly above a button, then the “2nd” button must be pressed first. For example, to enter the following syntax you must press the “2nd” button before the “SIN” button to access:

$\text{SIN}^{-1} \rightarrow 0.25 \rightarrow \text{ENTER}$

As you can see, it’s much easier to read the short-cut notation above than to read its wordy description.

So, are you ready to try some new techie stuff? Let’s get started!



# Algebra

## ***Horizontal and Vertical Linear Equations***

Let's start with lines. Lines are the simplest of all graphing equations. Sometimes linear equations are called first-order equations, meaning the largest exponent in the equation is a one (1). Later we'll look at second-order equations.

The easiest linear equations to graph are perfectly horizontal or vertical lines. Vertical lines are those whose equations look like  $x = k$ , where  $k$  is any real number. Horizontal lines are those whose equations look like  $y = k$ , where  $k$  is any real number.

The following two graphs are examples of *vertical* lines:

Figure 1

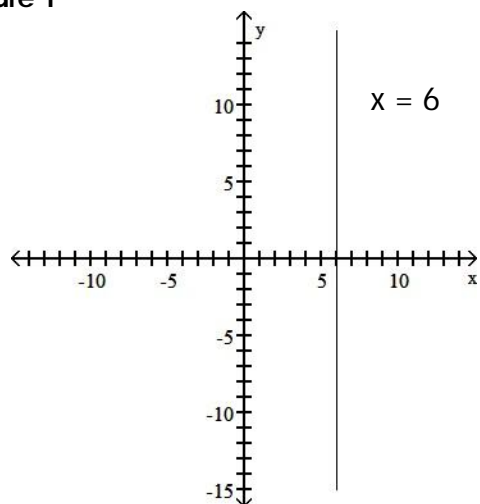
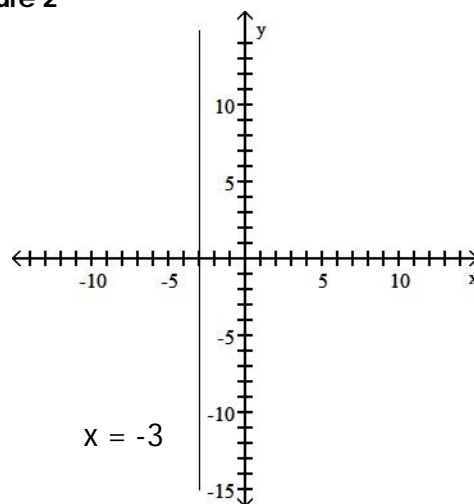


Figure 2



The equation of the vertical line in Figure 1 is  $x = 6$ , while the equation of the vertical line in Figure 2 is  $x = -3$ .



The following two graphs are examples of *horizontal* lines:

Figure 3

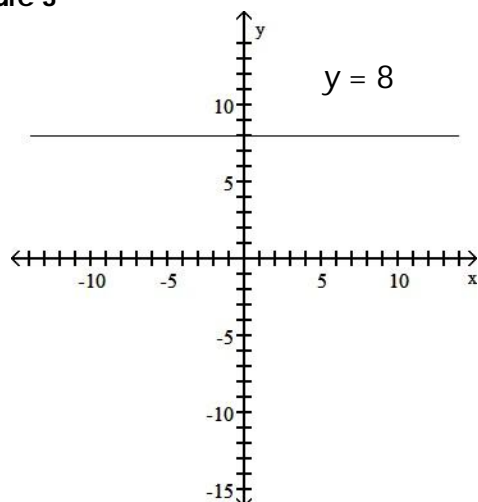
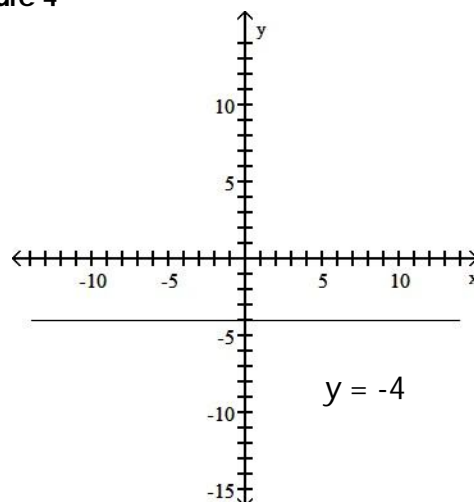


Figure 4



The equation of the horizontal line in Figure 3 is  $y = 8$ , while the equation of the horizontal line in Figure 4 is  $y = -4$ .

To graph these same equations using your TI-83/84 plus calculator, press the following buttons:

$Y= \rightarrow 8 \rightarrow \text{ZOOM} \rightarrow 6$

$Y= \rightarrow -4 \rightarrow \text{GRAPH}$

Note: make sure to use the negative button (-) located to the left of the ENTER button instead of the minus button to enter negative numbers. This is very important, and if not followed properly, will give erroneous answers or graphs.

Try graphing the following equations for practice:

$$Y = 1$$

$$Y = -10$$

$$Y = 0 \text{ (why doesn't this line show up?)}$$

Now try to graph equations with numbers bigger than 10 or smaller than -10, such as  $y = 18$ . When you graph equations like these, they do not show up on the screen. That's because the window size is too small and does not include the graph you are trying to display.

## TI WINDOW Settings



To adjust the window size, press the WINDOW button located near the top of the calculator. Move the cursor up and down, changing the settings to the following:

```
WINDOW
Xmin=-20
Xmax=20
Xscl=5
Ymin=-20
Ymax=20
Yscl=5
Xres=1
```

Wondering what these numbers or settings mean? Let me explain. The Xmin setting determines what the smallest x-value will be to show up on the far left of the graph, whereas the Xmax setting determines what the largest x-value will be to show up on the far right of the graph; likewise for the Ymin and Ymax values. So those four settings determine the new window size.

The Xscl and Yscl settings pertain to the number of “tick marks” you see along the x-axis and the y-axis in the graph. Since the largest value of x on the x-axis is set to 20, then there should be 4 “tick marks” located along the positive x-axis, since 20 divided by 5 equals 4. Likewise, there should be 4 “tick marks” located along the negative x-axis.

To see your new settings take effect, press the GRAPH button. Do you see the new window size? Feel free to change the WINDOW settings anytime you’d like to, especially if what you are trying to graph does not get displayed appropriately.

By the way, the WINDOW settings for X and Y do not always have to be the same values. You could have WINDOW settings that look like this:

```
WINDOW
Xmin=-10
Xmax=1
Xscl=2
Ymin=-20
Ymax=20
Yscl=4
Xres=1
```

## General Linear Equations

Now let’s graph linear equations that have a slope to them. Recall that linear equations that are perfectly horizontal have a slope of zero, and linear equations that are vertical have no slope at all. We say that vertical lines have a slope that is undefined, since we can not divide by zero in math.



$Y = 8$  is a horizontal line with slope = 0.

$X = -7$  is a vertical line with slope = undefined (or no slope).

So, if a line is neither horizontal nor vertical, then it either has a positive slope or a negative slope. Look at the two lines in Figure 5 and Figure 6. The line in Figure 5 has a positive slope since the line rises up, when your eyes glance across it from left to right. The slope in Figure 6 is negative since it falls down, when your eyes glance across it from left to right.

Figure 5

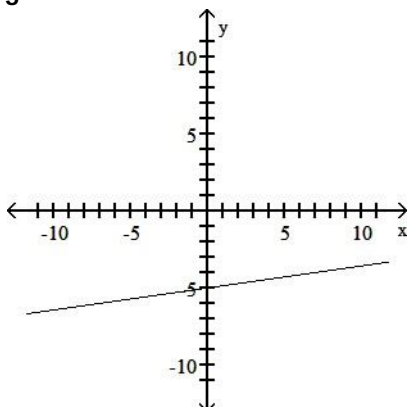
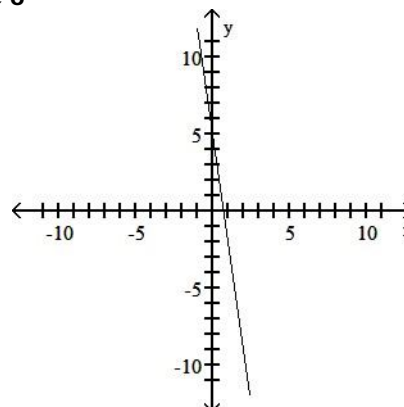


Figure 6



Perhaps you've seen the following equation plenty of times. It's called the *slope-intercept form* for an equation of a line.

$$y = mx + b$$

Recall that the slope in this equation is represented by the  $m$ , while the  $b$  represents the  $y$ -intercept (where the line crosses through the  $y$ -axis).

The equation of the line in Figure 5 is  $y = \frac{1}{7}x - 5$ .

The equation of the line in Figure 6 is  $y = -7x + 5$ .

Let's graph each of these separately on the calculator. Later I'll show you how to graph both of them at the same time.

First, set the window size to the following standard:

$$X_{\min} = -10$$

$$X_{\max} = 10$$

$$X_{\text{scl}} = 1$$

$$Y_{\min} = -10$$

$$Y_{\max} = 10$$

$$Y_{\text{scl}} = 1$$

The short-cut to do this is: **ZOOM  $\rightarrow$  6**





When you press the ZOOM button and choose option number 6:ZStandard, the above window settings are automatically set for you. Cool, huh?

Now enter the following into your calculator:

$$Y = \rightarrow 1 / 7 X - 5$$

The button for X is located near the upper left corner of the calculator. The button is labeled as "X,T,θ,n." Once you enter the equation into the first equation,  $Y_1=$ , press the GRAPH button. You will see the graph shown in Figure 5 on your calculator.

Let's graph the second equation. Start by pressing  $Y=$  and then CLEAR. This clears the first equation you entered into  $Y_1=$ . Now enter the following into your calculator:

$$-7X + 5$$

Note: that's a negative in front of the 7, not a minus sign.

Try graphing the following equations for practice.

$$Y = 3X - 4$$

$$Y = -1/4X + 2$$

$$X = Y - 2 \quad (\text{HINT: first make the equation look like } y = mx + b.)$$

$$-7X + Y = 0$$

$$X + Y = 20 \quad (\text{HINT: change the WINDOW size to see the graph.})$$

## Matrices

An often overlooked concept in high school math is matrices (plural form of "matrix"). A matrix is a rectangular table of numbers arranged in rows and columns. For example, here is a matrix that has 2 rows and 3 columns:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Its dimensions are 2 x 3.

Notice the square brackets [ ] around the matrix? Whenever you see those, you know you're working with a matrix. Every matrix has dimensions, which indicate how many rows and columns it has. Dimensions are always noted as ROWS x COLUMNS, never COLUMNS x ROWS.

Here are other matrices and their dimensions:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

dimensions: 1 x 4



$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad \text{dimensions: } 3 \times 2$$

The TI graphing calculator can handle matrices and matrix operations. First let's see how to enter a matrix into the calculator, then later we'll get to matrix operations using the calculator.

$$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

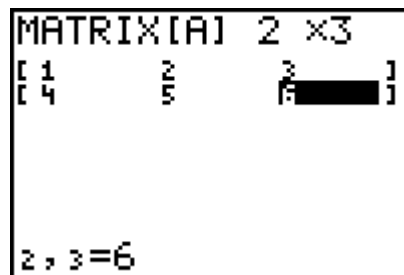
MATRIX → ►► → ENTER

You should now be in a mode to edit matrix [A]. We need to tell the calculator what the dimensions of the matrix are.

2 → ENTER → 3 → ENTER

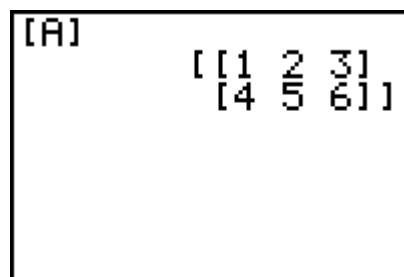
You should see the actual size of the matrix change as you press the ENTER button. Now, let's enter the values in each cell.

1 → ENTER → 2 → ENTER → 3 → ENTER → 4 → ENTER → 5 → ENTER → 6 → ENTER



We've now filled the matrix correctly. Press the QUIT button to clear the screen without clearing any data.

To see our new matrix, do the following:



MATRIX → ENTER → ENTER

So, to edit a matrix, remember to move your cursor to the EDIT button, then choose which matrix ([A], [B], [C], etc.) you want to update.

## Determinant of a Matrix



One operation performed on matrices is to determine its *determinant*. In the simple case of a 2 x 2 matrix, the determinant is found by doing the following:

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - cd$$

Let's find the determinant of [B], if  $[B] = \begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix} = 1 \cdot 6 - 2 \cdot 4 = 6 - 8 = -2$ . So, the determinant of [B] = -2. We can do the same thing using the calculator.

MATRIX → ►►▼ → ENTER → 2 ENTER → 2 ENTER  
 1 ENTER → 4 ENTER → 2 ENTER → 6 ENTER → QUIT  
 MATRIX → ► ENTER → MATRIX → ▼ ENTER → ENTER

Perhaps you're thinking that's a lot of work, punching all those buttons, when you can find the determinant of the matrix in possibly a shorter amount of time. This is true, if the dimensions of the matrix are relatively small. But, if the matrix is a 3 x 3 or larger, then using a calculator is so much easier. Calculating the determinant of a 3 x 3 matrix is beyond the scope of this lesson. Trust me, using a calculator is simpler.

## ***Solving Systems of Linear Equations***

Finally, we get to put everything we've done so far to better use — solving a system of linear equations. A system of linear equations is simply grouping two or more linear equations together and graphing them on the same coordinate plane. If they intersect each other, then the coordinate of that point is called the "solution" of the system. There are several different ways to solve systems of equations:

Graphing Method

Substitution Method

Elimination (or Addition) Method

Cramer's Rule (using matrices)

We can use our calculator to solve systems of equations using the graphing method and by Cramer's rule. The following is a system of equations that we'll solve using the graphing method.

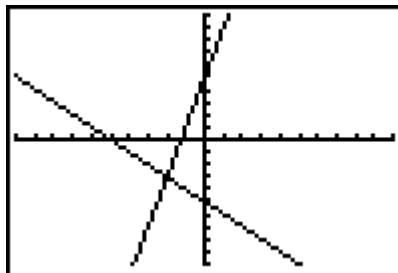
$$\begin{cases} y = 4x + 5 \\ y = -x - 5 \end{cases} \quad \begin{array}{l} \text{Enter both of these equations in your calculator. Make the} \\ \text{first one } Y_1 \text{ and the second one } Y_2. \end{array}$$

Y= → 4X + 5 → ▼ → -X - 5 → GRAPH



```

Plot1 Plot2 Plot3
Y1=4X+5
Y2=-X-5
Y3=
Y4=
Y5=
Y6=
Y7=
    
```



You should see both lines graphed at the same time. Can you determine where they intersect with each other? It may be a little difficult to see. One thing you can do is resize the WINDOW

to “zoom in” on the intersection. Change your WINDOW size to the following settings:

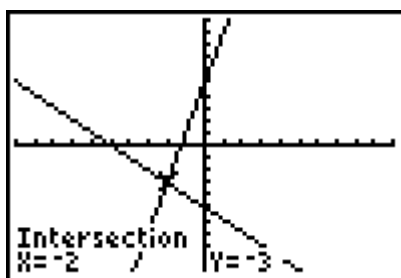
WINDOW → Xmin: -5 → Xmax: 5 → Ymin: -5 → Ymax: 5 → GRAPH

Can you see it better now? It looks like they intersect at (-2, -3). To be sure, here’s a pretty cool function built into the calculator.

```

2nd > CALC
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
    
```

CALC → 5:intersect → ENTER → ENTER → ENTER



At the bottom of the graph, you should see X = -2 and Y = -3. This confirms our answer! Awesome, huh?

Now for Cramer’s rule using our calculator. But first, to review what Cramer’s rule is:

$$X = \frac{\det["X \text{ matrix}"]}{\det["Coefficient \text{ matrix}"]}$$

$$Y = \frac{\det["Y \text{ matrix}"]}{\det["Coefficient \text{ matrix}"]}$$

What does this mean? Well, first let’s rewrite the two equations in our system so they look like this:

$$\begin{cases} 4x - y = -5 \\ x + y = -5 \end{cases}$$

Notice that in both equations x and y are on the left-hand side, with the constant on the right-hand side by itself.

Now, looking at the left-hand side of both equations, and isolating the coefficients, gives us a “coefficient matrix” of:



$$\text{Coefficient matrix} = \begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix}$$

If we replace the first column of this matrix with the “constant” column we get what we’ll call the “X matrix.”

$$\text{X matrix} = \begin{bmatrix} -5 & -1 \\ -5 & 1 \end{bmatrix}$$

Similarly, if we replace the second column of the coefficient matrix with the “constant” column we get what we’ll call the “Y matrix.”

$$\text{Y matrix} = \begin{bmatrix} 4 & -5 \\ 1 & -5 \end{bmatrix}$$

Finally, using Cramer’s rule to determine the value of  $x$ , we divide the determinant of the X matrix by the determinant of the Coefficient matrix. Let’s use our calculators, assigning [A] to the coefficient matrix, [B] to the X matrix, and [C] to the Y matrix.

MATRIX → ►► → ENTER → 2 ENTER → 2 ENTER  
 4 ENTER → -1 ENTER → 1 ENTER → 1 ENTER  
 MATRIX → ►►▼ → ENTER → 2 ENTER → 2 ENTER  
 -5 ENTER → -1 ENTER → -5 ENTER → 1 ENTER  
 MATRIX → ►►▼▼ → ENTER → 2 ENTER → 2 ENTER  
 4 ENTER → -5 ENTER → 1 ENTER → -5 ENTER → QUIT

Now let’s perform the operations of Cramer’s rule. To find the value of  $x$ :

MATRIX → ► ENTER → MATRIX → ▼ ENTER → ) ÷ →  
 MATRIX → ► ENTER → MATRIX → ENTER → ENTER

Did the calculator give you an answer of -2?

Do the same thing to find the value of  $y$ , except that it’s the calculation is  $\det([C]) / \det([A])$ . Did you get an answer of -3?



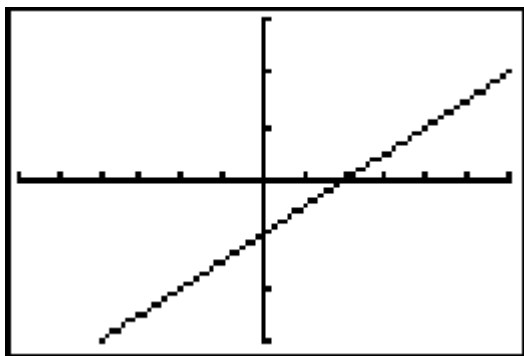
Name \_\_\_\_\_

Date \_\_\_\_\_

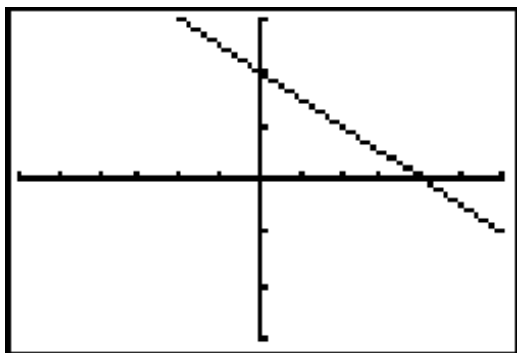
Practice: Linear Equations

Page 1

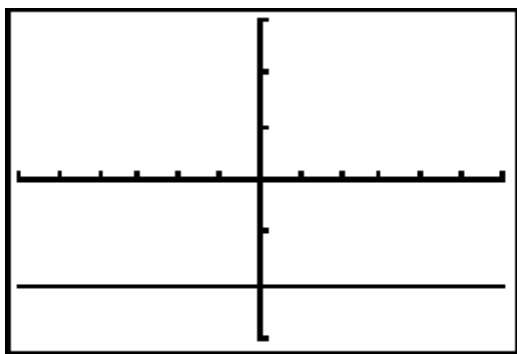
Determine the equation of the line of the following graphs. Write your answer in  $y = mx + b$  format on the provided line.



1. \_\_\_\_\_



2. \_\_\_\_\_



3. \_\_\_\_\_



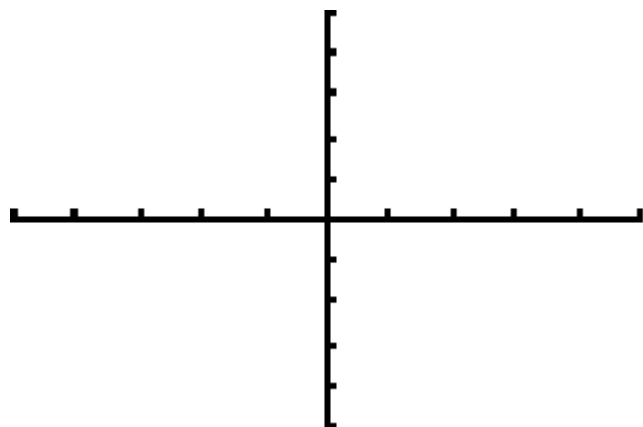
Name \_\_\_\_\_

Date \_\_\_\_\_

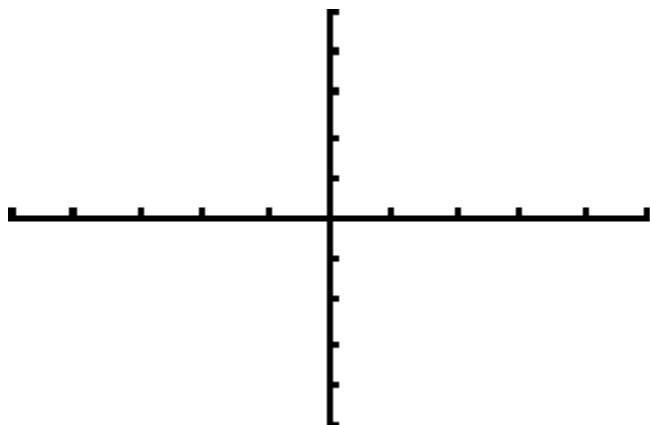
Practice: Linear Equations

Page 2

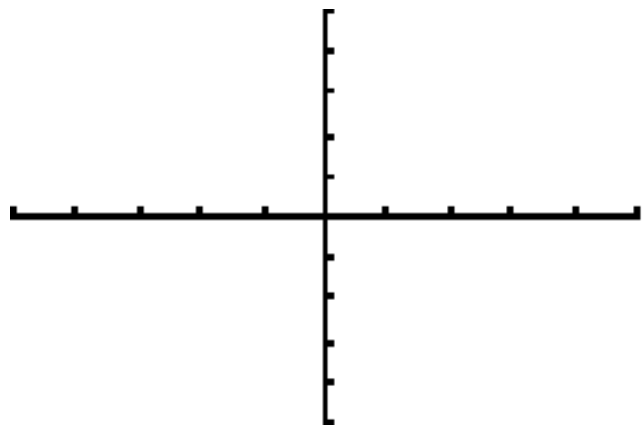
Graph the following equations on your graphing calculator and, as precisely as possible, sketch them onto the provided graphs. Use a straight edge for cleaner lines.



4.  $y = -\frac{3}{2}x - 3$



5.  $y = \frac{2}{3}x$



6.  $x = 4$



Name \_\_\_\_\_

Date \_\_\_\_\_

Practice: Systems of Equations

Page 1

Use a graphing calculator to solve the following system of equations using the indicated method. Write your answer as an ordered pair (coordinate).

1.  $x - 2y = 6$       Graphing  
 $x + 2y = 2$

Answer: \_\_\_\_\_

2.  $3x - 4y = 24$       Graphing  
 $y = -\frac{3}{2}x + 3$

Answer: \_\_\_\_\_

3.  $2x - y = 12$       Cramer's Rule  
 $3x + 2y = -3$

Answer: \_\_\_\_\_

4.  $3x - 7 = -5y$       Cramer's Rule  
 $5x + 4y = -10$

Answer: \_\_\_\_\_

5.  $4x + 3y = 1$       Method of your choice  
 $3x + 2y = 2$

Answer: \_\_\_\_\_ Method chosen: \_\_\_\_\_





Date \_\_\_\_\_

## Page 2

1. The average book at Barnes & Noble costs \$10 and at Borders Book Store costs \$8. If a group of 36 people purchased books from these two bookstores combined, spending a total of \$298, how many books from each bookstore were purchased?

- Two cars that were 450 miles apart traveled toward each other. They met after 5 hours. If one car traveled twice as fast as the other, what were their speeds?
- A newspaper advertised DVDs and CDs. Tracy went shopping and bought each of her seven nephews a gift, either a DVD or a music CD. Each DVD costs \$14.95 and each CD costs \$16.88, and she spent a total of \$114.30. How many DVDs and how many CDs did she buy?

Date \_\_\_\_\_

Page 3

- 



# Pre-Calculus

## *Polynomial Functions*

### Linear Functions

Linear Functions are exactly the same as linear equations covered in the previous Algebra lesson. Equations that have an independent variable and a dependent variable are considered functions. I will use the two words interchangeably throughout this unit. The following shows the difference in how each is written.

Linear equation:  $y = mx + b$

Linear function:  $f(x) = mx + b$

In this case,  $y$  and  $f(x)$  are exactly the same. The latter is read "f of x", where "f" is the name of the function.

Since linear equations and linear functions are essentially the same, graphically-speaking, let's concentrate on a different aspect of functions with regard to the TI graphing calculator.

Located above the GRAPH button is TABLE. The table function on the calculator lists both the independent variable ( $x$ ) and the dependent variable ( $y$ ). For example, if the linear equation or function we want to graph is  $y = 2x + 3$ , then the following table of independent and dependent variables would look like the following:

To see the same table on the calculator, first enter the equation into the calculator using the  $Y=$  button. Then, instead of pushing the GRAPH button, push the TABLE button located above it. Do you see the same table?

A button that coincides with TABLE is called TBLSET, located above WINDOW. Press the TBLSET button now, and you will see:

X	Y1	
0	3	
1	5	
2	7	
3	9	
4	11	
5	13	
6	15	
X=0		



```
TABLE SETUP
TblStart=1
ΔTbl=1
Indpt: Auto Ask
Depend: Auto Ask
```

TblStart indicates the first number displayed as the independent variable when the TABLE button is pressed.  $\Delta$  Tbl indicates the difference between each displayed independent variable. The default setting is 1. Finally notice that AUTO is selected for both the independent and dependent variables. This is also part of the default setting. If you prefer that the calculator asks (or prompts) you to enter

an independent value and in return provide you with its corresponding dependent value, then select ASK for the Indpt setting. Likewise, if you prefer that the calculator asks you to enter a dependent value and get in return its corresponding independent value, then select ASK for the Depend setting.

Change TBLSET to look like the following, then press TABLE and you should see this table:

```
TABLE SETUP
TblStart=-3
ΔTbl=.5
Indpt: Auto Ask
Depend: Auto Ask
```

X	Y <sub>1</sub>
-3	-3
-2.5	-2
-2	-1
-1.5	0
-1	1
-.5	2
0	3

Now, change TBLSET to look like the following, then press TABLE and enter the following independent (x) variables, one at a time, each followed by ENTER to get a different table:

```
TABLE SETUP
TblStart=-3
ΔTbl=.5
Indpt: Auto Ask
Depend: Auto Ask
```

X	Y <sub>1</sub>
7	17
-4	-5
0	3
16	35
-9.5	-16

## Quadratic Functions

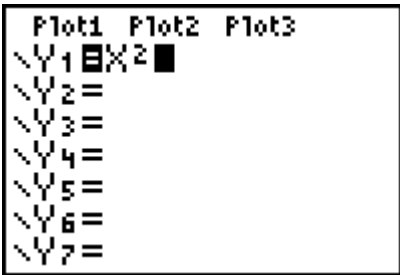
Quadratic functions are generally mistaken to be functions having to do with the number four. However, they have everything to do with the number two instead. These functions are easy to recognize because the largest exponent in a quadratic function is a two. Typically, the independent variable has the largest exponent, while the



dependent variable's exponent is a one (1). The following are examples of quadratic equations and quadratic functions. Can you tell which is which?

$y = x^2 + 3x - 4$        $f(x) = 8x^2 - 5$        $g(x) = -x^2 - x - 1$        $y = 3x^2 + 12x$

Let's start by looking at the simplest of all quadratic equations:  $y = x^2$ . This function's graph looks u-shaped. The u-shape could open up or open down, depending upon the sign of the leading coefficient (the coefficient of the  $x^2$  term). Using the TI calculator, let's enter the equation into Y=, set up the table using TBLSET, view the TABLE, then view its GRAPH.



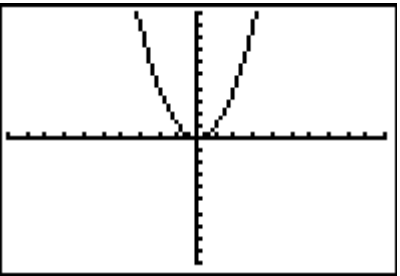
Y= → CLEAR (if necessary, to get rid of any previously entered functions)  
X → "x^2" (Sets  $Y_1 = X^2$ )  
TBLSET → TblStart = -3; ΔTbl = 1; Indpnt: AUTO;  
Depend: AUTO  
TABLE

The following table is displayed, which should make sense to you. If x equals 3, then when you square it, y = 9. Also, if x equals -3, and x is squared, then you still get y = 9.

X	Y1	
-3	9	
-2	4	
-1	1	
0	0	
1	1	
2	4	
3	9	

X=-3

Finally, let's GRAPH the function. If you do not see the parabola correctly, try ZOOM → 6:ZStandard.



Using the quadratic equations/functions given above, try entering each one into the calculator, graphing them, and looking at their table data.  
Which ones open up/down? Which one is really skinny?

Here's one more to try. This time let's leave the quadratic equation in its factored form like this:  $y = (x - 4)^2 + 2$ . You've probably learned that when written in this manner, a quadratic function's vertex (minimum or maximum point) is rather easily detectable. The coordinate of this equation's vertex is (4, 2). And since it opens up, the vertex is a minimum.

WINDOW → Xmin: 0 → Xmax: 6 → Ymin: 0 → Ymax: 6  
Y= → CLEAR (clear  $Y_1$  if necessary)  
( X - 4 ) → "x^2" → + 2 → GRAPH

Can you see where the vertex is? It should be located at (4, 2). To verify this using the calculator's minimum feature, try the following:

GRAPH → CALC → 3:minimum



The calculator should now be displaying the graph. It should also be prompting you to enter the Left Boundary of the minimum point. Using the left arrow, move the blinking cursor far enough to the left so that the  $X =$  value at the bottom of the screen is a number smaller than 4. Press ENTER. It should now prompt you to enter the Right Boundary of the minimum point. This time, using the right arrow, move the blinking cursor far enough to the right so that the  $X =$  value at the bottom of the screen is a number larger than 4. Press ENTER. The next prompt is Guess? In other words, would you like the calculator to give you its best guess? Press ENTER one more time.

The screen shows the graph, with a blinking dot on the minimum point, whose estimated value appears on the bottom of the screen as  $X = 3.9999991$  and  $Y = 2$ . Perhaps your calculator does not show the same value for  $x$  as mine does, but either way, it's close enough to  $X = 4$ ,  $Y = 2$ .

The same thing can be done for a quadratic equation whose graph opens down, except that instead you should use the "maximum" option under CALC.

## Cubic Functions

The last category of polynomial functions in this lesson is cubic functions. These function's largest exponent is three. And although the right-half of their graphs look like a parabola's, the left-half of its graph is inverted. Try graphing the following cubic function using the TI calculator:

$$y = x^3$$

Note: use the ^ button to get the cubic exponent, or press MATH and choose option 3.

Now that you are getting a better grasp of graphing polynomial equations, go ahead and get creative. Try graphing these polynomial functions. Change the WINDOW size if necessary to properly see the graph.

$$y = x^3 + 2x^2 - 7 + 3$$

$$y = x^4 - 2x^3 - x^2 - 1$$

$$y = -\frac{1}{4}x^5 + 3x$$

$$g(x) = x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$$

## Rational Functions

In math the term "rational" can be associated with fractions. The two are similar. So, rational functions are functions that have fractions in them. In fact, rational functions are functions with polynomial functions in both the numerator and denominator of a fraction.



The most important thing to keep in mind when dealing with rational functions is to avoid values of the independent variable that would make the denominator equal to zero. Remember, division by zero in math is "undefined."

Look at the following rational function, the simplest of all rational functions, and ask yourself what value for  $x$  would make the denominator zero.

$$y = \frac{1}{x}$$

This equation (function) would be undefined if  $x = 0$ . So when we graph

the equation, notice what happens to the graph as  $x$  approaches zero, both from the positive side and negative side.

Y= → CLEAR (if necessary) → 1 / X → ZOOM → 6:ZStandard

To see the graph better, change the WINDOW size to

WINDOW → Xmin: -5 → Xmax: 5 → Ymin: -5 → Ymax: 5 → GRAPH

Better yet, let's look at the actual values of the independent and dependent variables ( $x$  and  $y$ ) in the TABLE format of the function.

TABLE → scroll up or down until you see 0 in the X column

Do you notice what the value of  $Y$  equals when  $X$  equals zero? My calculator shows "ERROR," meaning, in this case, "undefined." The term that is used in pre-calculus for such a case as this is "asymptote." An asymptote is an invisible line that the actual graph approaches, but never quite reaches.

Let's graph other rational equations. Some graphs will have multiple asymptotes, others will have only one asymptote (like we just saw), and still others have no asymptotes but are still considered rational equations.

HINT: when entering rational equations into the calculator, it's a good idea to use parentheses to separate the numerator and denominator. In other words, wrap the entire numerator in  $()$ , then press the  $\div$  button, followed by the entire denominator wrapped in  $()$ .

$$f(x) = \frac{x}{x^2 - 9} \qquad g(x) = \frac{x+3}{x^2 - 9} \qquad h(x) = \frac{x+3}{x^2 + 9}$$

You should notice that  $f(x)$  has two asymptotes,  $g(x)$  has one, and  $h(x)$  has no asymptotes. To see the graph of  $h(x)$  better, try setting the WINDOW size to:

WINDOW → Xmin: -5 → Xmax: 5 → Ymin: -0.5 → Ymax: 0.5 → Yscl: 0.1



## Exponential Functions

So far the functions we've looked at have had the independent variable located in the base form:  $x$ ,  $x^2$ ,  $x^3$ ,  $x^5$ . Exponential functions reverse this — the independent variable is now located in the exponent and the base is a constant. For example, instead of a function looking like this:

$$f(x) = x^2 \quad \text{now it looks like this:} \quad f(x) = 2^x$$

Notice that the independent variable ( $x$ ) is now in the exponent and not the base. This is the definition of an exponential function.

Plugging some values for  $x$  into our new exponential function and seeing what the dependent variable becomes gives us a good idea of what the graph looks like.

$x$	$2^x$	Try this on your calculator: $Y = \rightarrow 2 \wedge X$ (the $\wedge$ button is located underneath the CLEAR button)
-3	1/8	
-2	1/4	
-1	1/2	Set your WINDOW to: WINDOW $\rightarrow$ Xmin: -5 $\rightarrow$ Xmax: 5 $\rightarrow$ Xscl: 1 $\rightarrow$ Ymin: -1 $\rightarrow$ Ymax: 10 $\rightarrow$ Yscl: 1 $\rightarrow$ GRAPH
0	1	
1	2	
2	4	Do you see that the graph rises to the right and drops close to the x-axis to the left? This type of function is called "exponential growth." Also notice that the function goes through the point (0, 1). That is, when the exponent is 0, the value of the function is one, no matter what the base is.
3	8	

To see an example of "exponential decay," try graphing the following function:

$$f(x) = 2^{-x} \quad Y = \rightarrow 2 \wedge (-X) \rightarrow \text{GRAPH}$$

Now, the graph rises to the left and falls to the right.

There are many applications of exponential growth and decay in the real world. We'll explore several of them in the exercise set at the end of the lesson.

## Logarithmic Functions

Most students seem to grasp the functions we've dealt with so far. However, when the topic of logarithms is introduced, students in general seem baffled, and rightly so. Logarithms are not very intuitive. What might help is to think of logarithms (*log* or *logs* for short) as inverses of exponential functions. Do you recall the relationship between inverse functions?





Better yet, here's another way to introduce logarithms. Begin with showing students how to solve a polynomial equation. When given a polynomial equation to solve, such as  $y = x^2 - 4$ , the right-hand side can be factored into  $y = (x + 2)(x - 2)$ , and if the equation is set equal to zero instead of  $y$ , then the two values of  $x$  that solve the equation are  $x = 2$  and  $x = -2$ . This concept is taught in higher Algebra courses.

However, what if the variable you are solving for in an equation is not in the base, but rather in the exponent? Wow! Are we supposed to solve the exponential equation for  $x$ , as was done in the previous example? Well, that's where logs come in. A logarithm is simply a way to isolate the independent variable  $x$ , from its exponent location. Thus, the following relationships must be memorized by math students, so that they can fully understand how much exponential functions and logarithmic functions rely on each other.

Exponential function:  $y = b^x$

Logarithmic function:  $x = \log_b y$

With these two equations in mind, you can now see how the following exponential equation can be rewritten into a logarithmic equation:

Exponential function	Converts to →	Logarithmic function
$y = 2^x$		$x = \log_2 y$

Notice that the exponent,  $x$ , is now isolated and can be solved for. Once the relationship between exponential and logarithmic functions is realized, understanding and working with logarithmic functions is easy and simple.

Keep in mind that when the LOG button is used on the calculator, the base is automatically understood to be 10. Also, when the LN button is used, the base is automatically understood to be  $e$  (the natural log). The logarithmic function with base 10 is called **common logarithmic** function. And the logarithmic function with base  $e$  is called the **natural logarithmic** function.

This time, let's graph both an exponential function and a logarithmic function. That way we'll see the inverse relationship between the two. Here are the two functions:

Exponential function:  $y = 10^x$

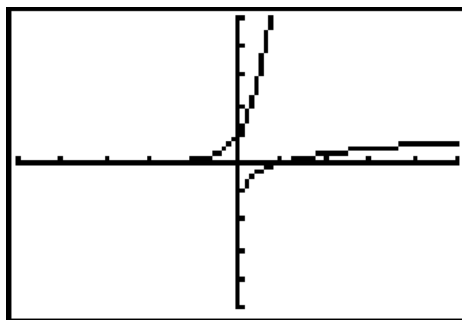
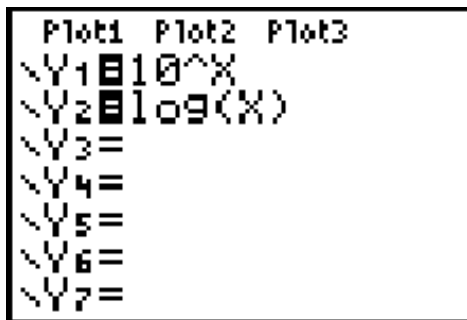
Logarithmic function:  $y = \log x$

Enter the exponential function first into  $Y_1$  then GRAPH it. Next, enter the log function into  $Y_2$  then GRAPH it.

WINDOW → Xmin: -5 → Xmax: 5 → Xscl: 1 → Ymin: -5 → Ymax: 5 → Yscl: 1  
Y= → 10 → ^ → X → GRAPH



Y= → ▼ → LOG → X → ) → GRAPH



Notice that one function is a reflection of the other over the line  $y = x$ .

## Miscellaneous Functions

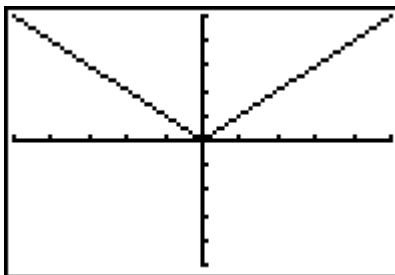
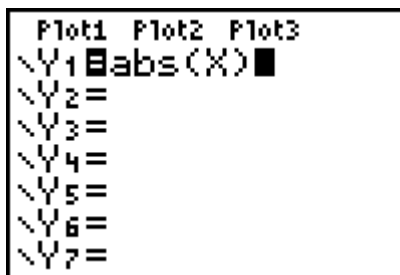
These last two functions we will graph are relatively popular ones, although not as popular as the previous functions. The absolute value function is easily recognized by its v-shaped graph. And the square root function sort of looks like a quadratic function lying on its side.

### Absolute Value Function

The absolute value function is a fun one. Whatever independent value you give the function, it returns the absolute value of that value, meaning the answer is always positive. Keep in mind that absolute value really means the "distance a number is away from zero." Thus whether the independent value is positive or negative, the answer is positive, since distance is always positive.

To get to the absolute value button on the calculator, first press the MATH button, then move the cursor to the right once, so that NUM is highlighted at the top of the screen. You should now see "abs(" as the first option. Remember this when entering the following,  $y = \text{abs}(x)$ :

Y= → MATH ► → ENTER → X ) → GRAPH

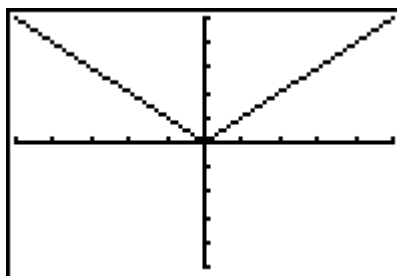




After further observation, do you notice that the right-hand side of the graph is exactly the same as  $f(x) = x$ , whereas the left-hand side of the graph is exactly the same as  $f(x) = -x$ ? So, another way to graph the absolute value function is like this:

```

Plot1 Plot2 Plot3
Y1=-X(X<0)
Y2=X(X≥0)
Y3=
Y4=
Y5=
Y6=
Y7=
    
```



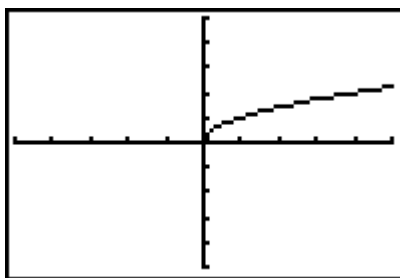
Here, I forced the calculator to only graph the domain of independent variables I specified. For the first function ( $Y_1$ ) only values of  $x$  less than zero, or negative values, were used. For the second function ( $Y_2$ ) only values of  $x$  greater than or equal to zero, non-negative values, were used.

To access symbols such as *greater than* ( $>$ ) or *less than* ( $<$ ), press these buttons:

2<sup>nd</sup> → MATH → option 3 or 4 or 5 or 6

## Square Root Function

The square root function,  $y = \sqrt{x}$ , only works with independent variable values that are positive or zero. Recall that the square root of a negative number is not a real number. Therefore the graph of the square root function only appears on the right (positive) side of the Cartesian plane. These functions are created using the  $\sqrt{\phantom{x}}$  button on the calculator.

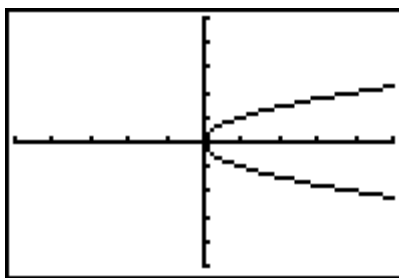
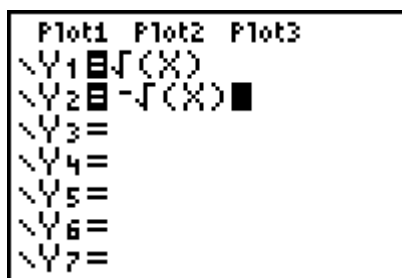


$Y = \sqrt{\phantom{x}} \rightarrow X \rightarrow \text{GRAPH}$

Again, don't forget to include the closing parenthesis.

To show you that the square root function looks like a quadratic function lying on its side, try the following: Leave the positive square root function you recently entered in  $Y_1$ , and enter this next function into  $Y_2$ , graphing both of these equations at the same time.

$Y = -\sqrt{\phantom{x}} \rightarrow X \rightarrow \text{GRAPH}$



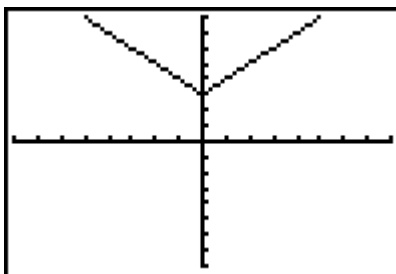
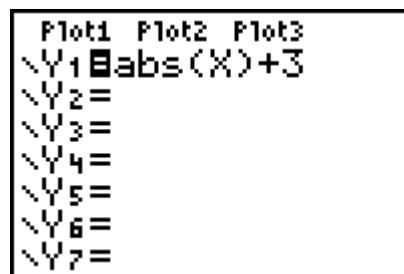
Don't the graphs of these two functions look like a quadratic function lying on its side?

## Transformations of Functions

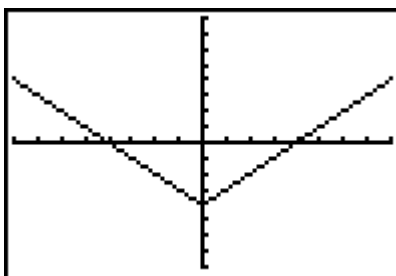
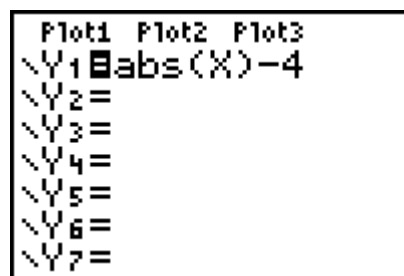
Let's do one more thing before we leave these functions. Let's translate them. In other words, observe what happens to the graph when we shift the function up or down, left or right. We could even reflect the graph over the x-axis or the y-axis.

First, using the absolute value function, we'll shift it vertically. To do that we'll rewrite the function by adding or subtracting a constant like this:

$$f(x) = |x| + 3 \quad \text{vertical shift up 3}$$

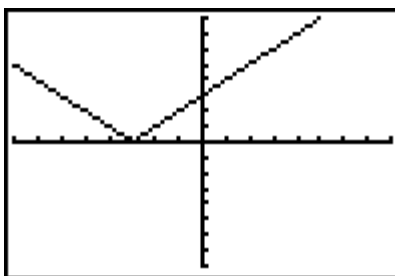
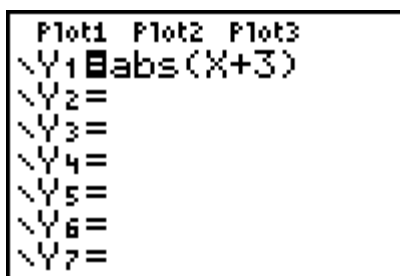


$$f(x) = |x| - 4 \quad \text{vertical shift down 4}$$



For horizontal shifting we add or subtract a constant directly affecting the independent variable  $x$ , inside the function, like this:

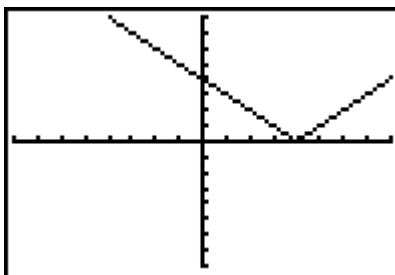
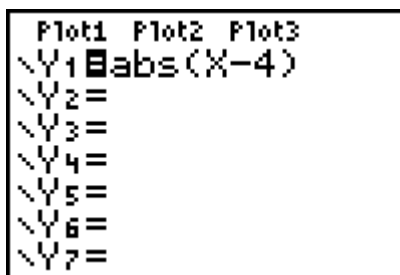
$$f(x) = |x + 3| \quad \text{horizontal shift left 3}$$



Interestingly enough, horizontal shifting is counter-intuitive, isn't it? One would expect that if we add 3 to  $x$ , we'd shift the function to the right, since positive numbers are to the right of zero, and negative numbers are to the left of zero. It turns out to be completely the opposite of what we would expect.

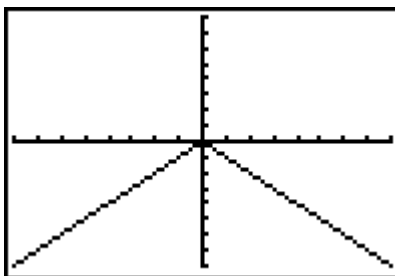
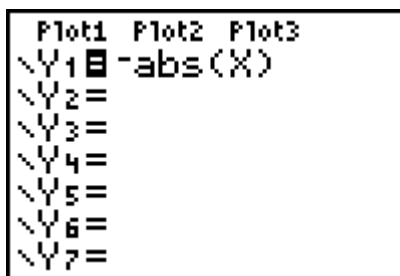
$$f(x) = |x - 4|$$

horizontal shift right 4



Now let's try to reflect (or flip) the function over the  $x$ -axis. To do that, observe what's different in the original absolute value function:

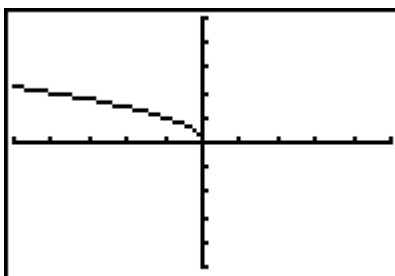
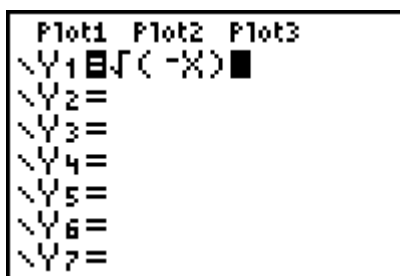
$$f(x) = -|x|$$



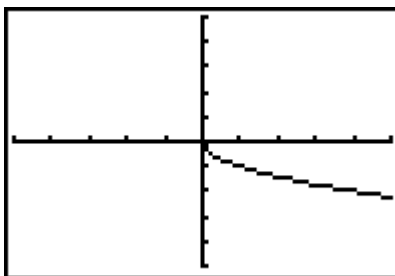
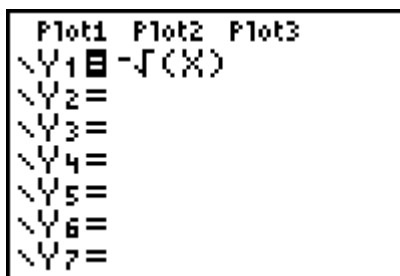
A reflection over the  $y$ -axis is not noticeable using the absolute value function, so let's try a different function, such as the square root function. Do you remember what that function looks like? By itself, it appears to be the upper portion of a quadratic function lying on its side.

So, to reflect it over the  $y$ -axis, we negate the independent variable  $x$ , directly:

$$y = \sqrt{-x}$$



Reflection over y-axis.



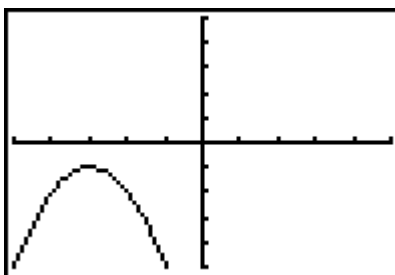
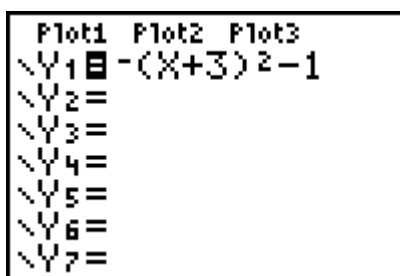
Reflection over x-axis.

These properties of transformations can be applied to any of the functions mentioned so far – linear, quadratic, cubic, rational, exponential, logarithmic, absolute value, and square root. Later you will see that they even apply to trigonometric functions.

Multiple properties of transformations can be used at the same time in a function. This could, for example, shift a function to the right, shift it down, and reflect it, all at the same time. Let's try one like that.

$$y = -(x+3)^2 - 1$$

Here is a quadratic equation whose vertex has been shifted to the left 3 and down 1. Can you see where that comes from in the function? Finally, the function is also reflected over the x-axis, as there is a negative sign out in front of the entire function. Let's plug it into the calculator and graph it. Be careful to use a negative sign out front and a minus sign before the number one.



## Summary of Transformations



Description	Change to Function	Change to Graph
Vertical Shift	$y = f(x) + c$	Shifts up
	$y = f(x) - c$	Shifts down
Horizontal Shift	$y = f(x + c)$	Shifts left
	$y = f(x - c)$	Shifts right
Reflection about the x-axis	$y = -f(x)$	Flips the graph over the x-axis
Reflection about the y-axis	$y = f(-x)$	Flips the graph over the y-axis



Name \_\_\_\_\_

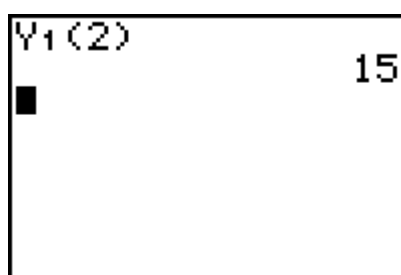
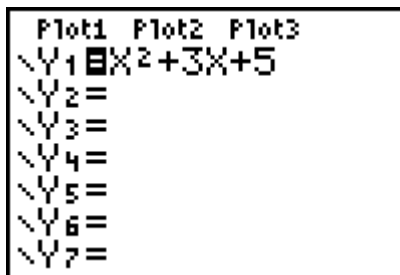
Date \_\_\_\_\_

Practice: Polynomial Functions

Page 1

Evaluating a function using a graphing calculator is simple to do. First, enter a given function into one of the numbered Y= equations, then enter the value at which you wish to evaluate into the function.

For example: Given  $f(x) = x^2 + 3x + 5$ , evaluate  $f(2)$ .



Thus  $f(2) = 15$ . The buttons to push to get the calculator to execute this evaluation go as follows:

VARs → Y-VARS → Function...  
→ Y<sub>1</sub> → (2) → ENTER

Evaluate the following polynomial functions:

1.  $f(x) = 4x + 5$ ;  $f(6) = ?$  \_\_\_\_\_

2.  $g(x) = x^2 + 2x + 3$ ;  $g(-1) = ?$  \_\_\_\_\_

3.  $h(x) = x^4 - x^2 + 1$ ;  $h(2) = ?$  \_\_\_\_\_

4.  $f(r) = \sqrt{r+6} + 3$ ;  $f(10) = ?$  \_\_\_\_\_

5.  $f(x) = \frac{4x^3 + 1}{x^3}$ ;  $f(-2) = ?$  \_\_\_\_\_

6.  $g(x) = \frac{|x+3|}{x+3}$ ;  $g(-5) = ?$  \_\_\_\_\_





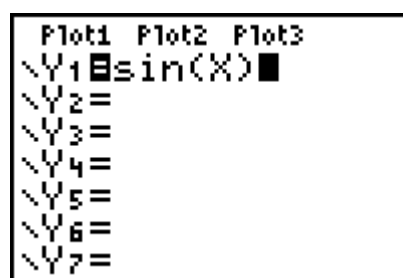


# Trigonometry

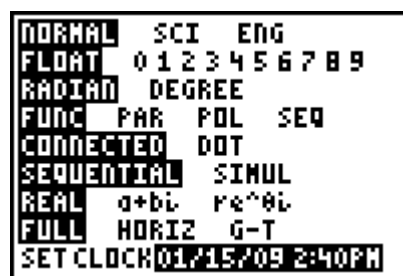
## *Graphs of Sine and Cosine*

Now we get into a slightly different kind of function. The sine, cosine, and tangent functions differ from the previous functions in that they are cyclical. It is especially noticeable with the sine and cosine functions. These three functions “repeat” themselves over and over again throughout their graphs over a subset of their domain. The subset I am referring to is called the function’s **period**. The period of a trig (short for trigonometric) function is the interval of its domain in which the graph of the function has a pattern that repeats itself. Let’s look at a simple example:

$$y = \sin x$$



Before we graph it, there are a couple of things we should do first. Make sure your calculator is set to the RADIAN mode and not the DEGREE mode. To check this, push the MODE button located near the top of the calculator. You should see this screen:



Make sure the third option down from the top is set to “RADIAN” and not “DEGREE”. This will make your trig graphs look much better.

One more thing before we graph. Let me show you what your window size should be set to:



```

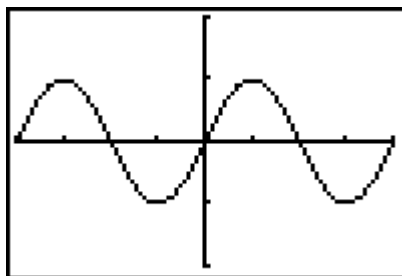
WINDOW
Xmin=-6.283185...
Xmax=6.2831853...
Xscl=1.5707963...
Ymin=-2
Ymax=2
Yscl=1
Xres=1
    
```

Notice that the decimals in this screen have a “...” at the end of them. This means that the decimals continue on, forever. So, instead of typing in the decimals as we see them, let’s enter the exact values, like this:

$$\begin{aligned} X_{\min} &= -2\pi \\ X_{\max} &= 2\pi \\ X_{\text{scl}} &= \pi/2 \end{aligned}$$

You will notice that once you enter in these irrational numbers, the calculator will automatically change the values into their decimal equivalents for you. Cool, huh?

Now you’re ready to graph the sine function. Hit the GRAPH button and see what it looks like:



Its repeating pattern is obvious. The period of this function is measured from consecutive peak to peak, or trough to trough, or some other combination of corresponding points. Also, notice that the highest point that the sine function reaches is one (1), and, likewise, its minimum point is a negative one (-1). This is called the function’s **amplitude**.

So, the basic sine function looks like this:

$$y = A \sin(Bx - C) + D$$

where:

- A is the amplitude of the function (maximum & minimum points),
- B is used to determine the period of the function, like this:
  - $\text{period} = 2\pi / B$
- C is used, along with B, to find what’s called the phase shift of the function, like this:
  - $\text{phase shift} = C / B$



- $D$  shifts the function vertically, up or down

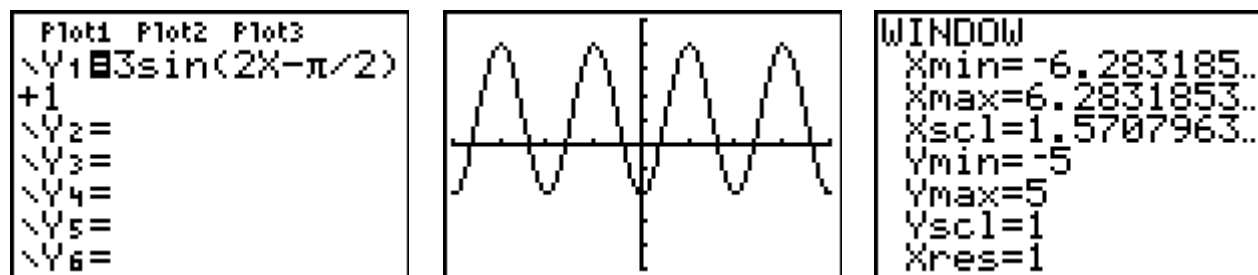
Here's an example that uses all of the above properties:

$$y = 3\sin\left(2x - \frac{\pi}{2}\right) + 1$$

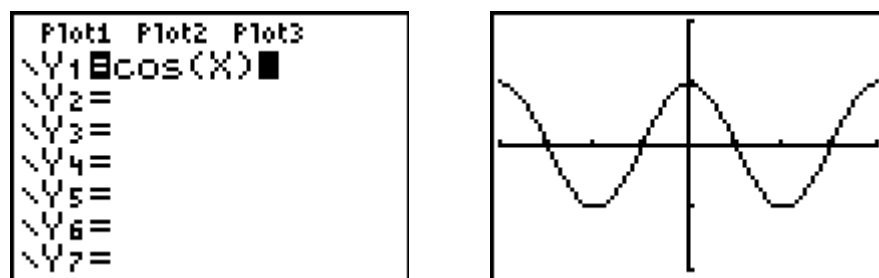
Knowing how the properties work, we can pretty much determine what the graph of this function looks like. First, determine the values of  $A$ ,  $B$ ,  $C$ , and  $D$ . Next, plug them into our properties:

- $A = 3$ , so our amplitude is 3. Meaning, the most this function will rise or fall from its center is 3 places.
- $B = 2$ , so the period of this function is  $2\pi/2$ , which when simplified equals  $\pi$ . The standard (normal) sine function has a period of  $2\pi$ . Thus this function will repeat its pattern in half the interval, or twice as fast.
- $C = \pi/2$ , so the phase shift of the function is  $(\pi/2)/2$ , which when simplified equals  $\pi/4$ . The standard (normal) sine function crosses the  $x$ -axis at 0. But this function moves that intercept  $\pi/4$  radians (45 degrees) to the right. (Recall the rules for horizontal shifting in the Transformation of Functions section.)
- $D = 1$ , so the function is shifted up, vertically, by one place.

Here's what it looks like:



What do you think? If you're wondering about the cosine function, the same rules or properties apply to it, too. Take a look at the standard cosine function:



Notice that the cosine function has a value of 1, when the independent variable,  $x$  equals 0. But, the graph is exactly the same as the sine function. This means that the cosine function is essentially the sine function with a phase shift of  $\pi/2$  (90 degrees) to



# Techie Math

the left. Try it. Graph the following function and compare it to the standard cosine function's graph:

```
Plot1 Plot2 Plot3
Y1=sin(X+π/2)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

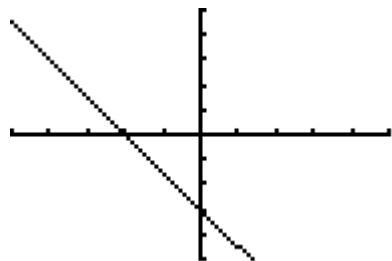




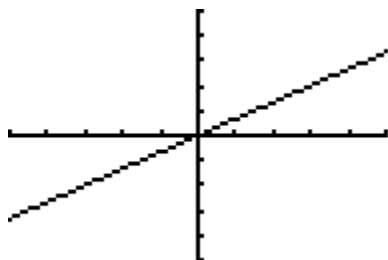
# Answers to Practice Problems:

## page 14

1.  $y = \frac{1}{2}x - 1$
2.  $y = -\frac{1}{2}x + 2$
3.  $y = -2$
- 4.



5.



6. A vertical line through 4 on the x-axis.

## page 16

1. (4, -1)
2. (4, -3)
3. (3, -6)
4. (-6, 5)
5. (4, -5)

## page 17

1. 5 books from B&N; 31 books from Borders
2. faster car: 60 mph; slower car: 30 mph
3. DVDs: 2; CDs: 5
4. 80L of 40% solution; 40L of the 70% solution
5. 30 barrels at \$40 per barrel; 20 barrels at \$60 per barrel
6. boat : 10mph; current: 2 mph

## page 32

1. 29
2. 2
3. 13
4. 7
5.  $31/8$
6. -1



# Resources

## ***Unit Resources:***

*Algebra & Trigonometry*; 3<sup>rd</sup> edition; Blitzer; Pearson Prentice Hall Publishing; © 2007.

*Introductory Algebra*; 8<sup>th</sup> edition; Lial, Hornsby, & McGinnis; Pearson Addison Wesley Publishing; © 2006.

<http://mathforum.org/dr.math/>

## ***Online Resources:***

- <http://www.algebrahelp.com/>
  - Algebrahelp.com is a collection of lessons, calculators, and worksheets created to assist students and teachers of algebra.
- <http://www.middleweb.com/CurrMath.html>
  - This website has multiple links within it, pointing you to different websites that offer help with math, from lesson plans to math explanations. Fantastic resource!
- <http://www.coolmath.com/>
  - Games, puzzles, math dictionary. It's all here and really fun!
- <http://www.qcc.mass.edu/mathsupport/arith.pdf>
  - Fractions, decimals, & percents explained, with examples & answers to several problems.
- <http://mathforum.org/dr.math/>
  - Have a math-related question? Ask Dr. Math. At this website there's bound to be an answer to your question!
- <http://www.sparknotes.com/math/>
  - Free study guides for various math topics.
- <http://www.manatee.k12.fl.us/sites/elementary/palmasola/mathlabtutorials.htm>
  - Fun explanations for mathematical thinking.





Online Resources: (cont.)

- <http://www.multcolib.org/homework/mathhc.html>
  - Another website chuck-full of math help and resources.
  - Want to know how an abacus works? Need geometry flash cards? It's here!
- <http://www.pbs.org/teachers/math/>
  - Lesson plans for multiple topics and grades
- <http://www.edu4kids.com/>
  - Click to the left-hand side on the link for "Math General" for flash cards and timed games.
- <http://www.mathsisfun.com/>
  - More fun with math explanations and games.