# Rational Numbers <br> Subtracting Rational Numbers <br> Lesson Plan 

I. Topic: Subtracting Rational Numbers
II. Goals and Objectives:
A. The student will learn how to subtract rational numbers with the same signs.
B. The student will learn how to subtract rational numbers with different signs.
C. The student will evaluate expressions using the subtraction properties.
D. The student will use the numbers line to subtract rational numbers.
E. The student will learn how to subtract terms with variables.
III. Southern Union Mathematics Standards:

1. CM.2.1

Concepts (number sense, algebraic and geometric thinking, measurement, data analysis and probability)
2. AL.2.2

Problem-solving skills (explore, plan, solve, verify.)
3. PA.3.3

Perform calculations with and without technology in life situations.
4. PA.4.2

Identify numbers and relationship among numbers.
5. PA.5.1

Use and evaluate expressions involving variables
IV. Materials:
A. Whiteboard with dry-erase markers (Blackboard with chalk could also be used.)
B. Ruler
C. Pencils
F. Subtracting Rational Numbers' Worksheets (Practice Worksheet, Quiz Worksheet)
V. Presentation Outline:
A. Introduction: "Subtracting Rational Numbers"
B. Subtracting rational numbers of the same signs Examples
C. Subtracting rational numbers of different signs Examples
D. Simplifying expressions Examples
E. Real life application

Examples
VI. Presentation:
A. Presentation Notes
B. Power Point Presentation
VII. Independent Practice: Subtracting Rational Numbers Worksheet
A. Class work: \# 1-49 Odds
B. Homework: \#2-50 Evens
C. Due 2 days from given day. Allow students to complete those questions which they did not complete in class.
VIII. Topic Assessment: Subtracting Rational Numbers' Quiz
A. Answer questions from homework.
B. 15-Question Quiz: 15 - 25 minutes


## Rational Numbers



Subtracting rational numbers can be a very simple process. The easiest rule to understanding rational numbers is:

To subtract a rational number, simply ADD its OPPOSITE.
There are other rules which can also help us identify the subtraction problem and also change the subtraction to an addition problem:

Negative $\boldsymbol{-}$ Positive $=$ Negative

- $(-5)-3=-5+(-3)=-8$
- $-4-10=-14$
- $-0.5-0.5=-1$
- $-3 / 4-(1 / 2)=-3 / 4-1 / 2=-5 / 4$

Positive $\boldsymbol{-}$ Negative $=$ Positive + Positive $=$ Positive

- $5-(-3)=5+3=8$
- $13-(-7)=13+7=20$
- $3 / 4-(-1 / 2)=3 / 4+1 / 2=5 / 4$
- $(7.7)-(-3.3)=(7.7)+3.3=11$

Negative $\boldsymbol{-}$ Negative $=$ Negative + Positive $=$ Use the sign of the larger number and subtract.

- $(-5)-(-3)=(-5)+3=-2$
- $(-6)-(-2)=(-6)+2=-4$
- $(-7.7)-(3.3)=(-7.7)+3.3=-4.4$
- $-3 / 4-(-1 / 2)=-3 / 4+1 / 2=-1 / 4$

One way to subtract rational numbers is using the number line.


Remember: When subtracting rational numbers on the number line, go to the left, starting from the first number given.

For example: $3-5=-2$
That is because we started at the number 3 on the number line.
Then we moved to the left 5 times. This will land us at the number -2 .

## Solve the following problems:

Problem: The temperature in Anchorage, Alaska was $8^{\circ} \mathrm{F}$ in the morning and dropped to $5^{\circ} \mathrm{F}$ in the evening. What is the difference between these temperatures?

Solution: We can solve this problem using integers. Using the number line below, the distance from ${ }^{+} 8$ to 0 is 8 , and the
 distance from 0 to 5 is 5 , for a total of 13 .
${ }^{+} 8-5{ }^{-}{ }^{+} 13$. The difference is 13 degrees.


Problem: The highest elevation in North America is Mt. McKinley, which is 20,320 feet above sea level. The lowest elevation is Death Valley, which is 282 feet below sea level. What is the difference between these two elevations?


Solution: When solving problems with large integers, it is not always practical to rely on the number line. Using integer arithmetic this problem becomes: $+20,320-282=$ ?

We need a rule for subtracting integers in order to solve this problem.
Remember: To subtract an integer, add its opposite.
The opposite of ${ }^{-282}$ is ${ }^{+} 282$, so we get: ${ }^{+} 20,320-{ }^{-} 282={ }^{+} 20,320+{ }^{+} 282={ }^{+} 20,602$

In the above problem, we added the opposite of the second integer, and subtraction was transformed into addition. Let's look at some simpler examples of subtracting integers:

Now let's solve the following examples. Use the given number line to help you find the solution.

Example 1:

$$
+5-{ }^{+} 2
$$



Step 1: $\quad$ The opposite of ${ }^{+} 2$ is 2.
Step 2:
Subtraction becomes addition.
Solution:
${ }^{+} 5-{ }^{+} 2={ }^{+} 5+2={ }^{+} 3$

Example 2: Find the difference between each pair of integers:


| Subtracting Rational Numbers |  |  |
| :---: | :---: | :---: |
| Subtract | Add The Opposite | Result |
| ${ }^{+} 9-{ }^{+} 4=$ | ${ }^{+} 9++^{-} 4=$ | ${ }^{+} 5$ |
| ${ }^{+} 7--^{-} 3=$ | ${ }^{+} 7+{ }^{+} 3=$ | ${ }^{+} 10$ |
| ${ }^{-} 1-^{+} 8=$ | ${ }^{-} 1+8=$ | -9 |
| ${ }^{-6}--^{-} 6=$ | ${ }^{-} 6+{ }^{+} 6=$ | 0 |

Notice that in each problem above, the first rational number remained unchanged. Also, do not confuse the sign of the rational number with the operation being performed.
Remember that:
$9+{ }^{+} 4=-5$ is read as Negative 9 plus positive 4 equals negative 5 .

Example 3: Find the difference between each pair of integers:


| Subtracting Rational Numbers |  |  |
| :---: | :---: | :---: |
| Subtract | Add The Opposite | Result |
| ${ }^{+} 7-{ }^{+} 10=$ | ${ }^{+} 7+{ }^{-} 10=$ | ${ }^{-} 3$ |
| ${ }^{+} 13-{ }^{-} 1=$ | ${ }^{+} 13+{ }^{+} 1=$ | ${ }^{+} 14$ |
| ${ }^{-} 8-{ }^{+} 6=$ | $-8+6=$ | ${ }^{-} 16$ |
| ${ }^{-} 1.4--2=$ | ${ }^{-} 1.4+{ }^{+} 2=$ | ${ }^{+} .6$ |

Remember: To subtract an integer, add its opposite.
Find the difference in the following integers:

| Subtraction Problem | Difference |
| :---: | :---: |
| $-9-(+12)$ |  |
| $15-(-7)$ |  |
| $-8-(+6)$ |  |
| $-14-(+9)$ |  |
| $-5-(+8)$ |  |
| $-13-(+13)$ |  |
| $32-(-12)$ |  |
| $-27-(+17)$ |  |
| $21-(+5)$ |  |
| $-10-(+11)$ |  |



## Simplifying and Evaluating Rational Expressions.

"Evaluation" mostly means "simplifying an expression down to a single numerical value." Sometimes you will be given a numerical expression where all you have to do is simplify; that is more of an order-of-operations kind of question.

In addition to simplifying expressions, there are times when we have to substitute the value of a variable before we can simplify the expression.

- First: substitute the value of the variable.
- Second: simplify the expression using PEMDAS.

Remember PEMDAS?

| PEMDAS - | Please | Excuse | My | Dear | Aunt | Sally |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Parenthesis | Exponent | Multiplication Division | Addition | Subtraction |  |

Usually the only hard part in evaluation is in keeping track of the minus signs. Keep parentheses liberally, especially when you're just getting started.

## Simplify and evaluate the following expressions:

- Evaluate $a^{2}-b$ for $a=-2, b=3, c=-4$, and $d=4$.

To find the answer, plug in the given values, being careful to use parentheses, particularly around the minus signs:

$$
(-2)^{2}-(3)=(4)-(3)=\mathbf{1}
$$

- Evaluate $a-c d$ for $a=-2, b=3, c=-4$, and $d=4$.

$$
(-2)-(-4)(4)=-2-(-16)=-2+16=14
$$

- Evaluate $(b-d)^{2}$ for $a=-2, b=3, c=-4$, and $d=4$.

Make sure not to "distribute" the exponent through the parentheses.
Exponents DO NOT distribute over subtraction!
$(b-d)^{2}$ is NOT the same as $b^{2}-d^{2}$. I must evaluate the expression as it stands:
$((3)-(4))^{2}=(-1)^{2}=\mathbf{1}$

- Evaluate $b^{2}-d^{2}$ for $a=-2, b=3, c=-4$, and $d=4$.

$$
(3)^{2}-(4)^{2}=9-16=-7
$$

Notice that this does not match the answer to the previous evaluation, pointing out again that exponents do not "distribute" the way multiplication does.

- Evaluate $b c^{3}-a d$ for $a=-2, b=3, c=-4$, and $d=4$.

$$
(3)(-4)^{3}-(-2)(4)=(3)(-64)-(-8)=(-192)-(-8)=-192+8=-184
$$

The most common "expression" you'll likely need to evaluate will be polynomials. To evaluate, you take the polynomial and plug in a value for $x$.

- Evaluate $x^{4}-3 x^{3}-x^{2}-6$ for $x=-3$.

$$
\begin{aligned}
& (-3)^{4}-3(-3)^{3}-(-3)^{2}-6 \\
& \quad=81-3(-27)-(9)-6 \\
& \quad=81-(-81)-9-6 \\
& \quad=\mathbf{1 6 2}-\mathbf{9}-\mathbf{6}=\mathbf{1 5 3}-\mathbf{6}=\mathbf{1 4 7}
\end{aligned}
$$

- Evaluate $3 x^{2}-12 x-4$ for $x=-2$.

$$
3(-2)^{2}-12(-2)-4=3(4)-(-24)-4=12+24-4=\mathbf{3 2}
$$

- Evaluate $y=4 x-3$ at $x=-1$.


$$
y=4(-1)-3=-4-3=-7
$$



Subtracting Integers
Student Practice Worksheet

Name $\qquad$ Date $\qquad$ Grade


Answer the following questions without a calculator. Use the number line for assistance if needed.
Add the following integers of the same signs:

1. $-94-(-27)$
2. $38-38$
3. $-47-(-28)$
4. $31-12$
5. $(-8)-(-79)$
6. $11-46$
7. $(-163)-(-239)$
8. (-150) - (-165)
9. $414-(+81)$

Add the following integers of different signs:
10. - $19-17$
11. $12-(-63)$
12. $-61-(5)$
13. $17-(-23)$
14. - 47-58
15. $7-(-31)$
16. - $154-75$
17. $398-(-300)$
18. -567-(0)

Add the following series of three or more integers:
19.
$(2)-(-6)-(-8)$
20. $-1-(-5)-(4)$
21. (-9) - (-7) - (-5)
22. (-8) - (1)-(9)
23. $7-(28)-(-10)$
24. (-14)-(80)-(-21)
25. (50) - (-90) - (-57)
26. (66) - (68) - (61)
27. $(-50)-(-86)-(111)$
28. (-93) - (93) - (-1)
29. (470)-(-366)-(180) 30. (-922)-(-747)-(551)

## Add the following mathematical expressions:

31. $69 w-18 w$
32. $12 \mathrm{x}-(-15 \mathrm{x})$
33. $-9 \mathrm{k}-7 \mathrm{k}$
34. $-18 p-25 p$
35. $-16 m-(-38 m)$
36. $6 n-(-8 n)-(-5 n)$

Evaluate each of the following expressions if $a=-3, b=-4$, and $c=5$ :
37. $\quad \mathrm{b}-(-8)$
38. $\mathrm{c}-\mathrm{b}$
39. $\mathrm{a}-\mathrm{c}-\mathrm{b}$
40. $a-(-12)$
41. $17-\mathrm{c}$
42. $\mathrm{c}-\mathrm{a}$
43. $\mathrm{a}-\mathrm{c}$
44. (-9) - (c) - (9)
45. $\mathrm{c}-\mathrm{b}-\mathrm{c}$

## Solve the following real world applications to simple addition problems:

Show your work and label your answer correctly.
46. Martha and Devin were playing Monopoly. Martha had $\$ 2,450$. She paid Devin $\$ 500$ because she landed on his property. How much money did Martha have left?
47. Kris baked 2 dozen cupcakes. Don ate five. How many cupcakes did Kris have left?
48. Dan went to lunch at $3: 25$. He returned from lunch at $3: 55$. How many minutes was he at lunch?
49. Miguel walked to his grandfather's house. He left home at $4: 55$. He arrived at his grandfather's at 5:20. How many minutes did it take Miguel to walk from his home to his grandfather's?
50. Ken goes to the store and buys a shirt that costs $\$ 5.99$. He gives the cashier a ten dollar bill. How much change will Ken receive? (Subtraction)

Subtracting Integers
Student Practice Worksheet
Answer Key
Name $\qquad$ Date $\qquad$ Grade $\qquad$
Answer the following questions without a calculator. Use the number line for assistance if needed.
Add the following integers of the same signs:

1. $-94-(-27)$
2. $38-38$
3. $-47-(-28)$
-67
0
4. $31-12$
5. $(-8)-(-79)$
6. $11-46$
19
71
$-35$
7. $(-163)-(-239)$
8. (-150) - (-165)
9. $414-(+81)$
76
15
333

Add the following integers of different signs:
10. - $19-17$
11. $12-(-63)$
12. $-61-(5)$
-36
13. $17-(-23)$
14. - 47-58
15. $7-(-31)$
40
$-105$
38
16. $-154-75$
17. $398-(-300)$
18. -567-(0)

Add the following series of three or more integers:
19.
(2) $-(-6)-(-8)$
20. $-1-(-5)-(4)$
21. (-9) - (-7) - (-5)

16
22. (-8) - (1)-(9)
23. $7-(28)-(-10)$
24. (-14)-(80)-(-21)
25. (50) - (-90) - (-57)
26. (66) - (68) - (61)
27. $(-50)-(-86)-(111)$
197
-63
-75
28. (-93)-(93)-(-1)
29. (470)-(-366) - (180) 30
(-922) - (-747) - (551)
-185
656
$-726$

## Add the following mathematical expressions:

| 31. | $69 \mathrm{w}-18 \mathrm{w}$ | 32. | $12 \mathrm{x}-(-15 \mathrm{x})$ | 33. |
| ---: | :---: | :---: | :---: | :---: |
|  | 51 w | $-9 \mathrm{k}-7 \mathrm{k}$ |  |  |
| 34. | $-18 \mathrm{p}-25 \mathrm{p}$ | 27 x |  | -16 k |
|  | 35. | $-16 \mathrm{~m}-(-38 \mathrm{~m})$ | 36. | $6 \mathrm{n}-(-8 \mathrm{n})-(-5 \mathrm{n})$ |
| -43 p |  | 22 m |  | 19 n |

Evaluate each of the following expressions if $a=-3, b=-4$, and $c=5$ :
37. $\quad \mathrm{b}-(-8)$
38.
c-b
39. $\mathrm{a}-\mathrm{c}-\mathrm{b}$
9
6
40. $a-(-12)$
41. $17-\mathrm{c}$
42. $\mathrm{c}-\mathrm{a}$
12
8
43. $\mathrm{a}-\mathrm{c}$
44.
$(-9)-(c)-(9)$
-23
45. $\mathrm{c}-\mathrm{b}-\mathrm{c}$
4

Solve the following real world application to simple addition problems:
Show your work and label your answer correctly.
46. Martha and Devin were playing Monopoly. Martha had $\$ 2,450$. She paid Devin $\$ 500$ because she landed on his property. How much money did Martha have left?

$$
\$ 1,950
$$

47. Kris baked 2 dozen cupcakes. Don ate five. How many cupcakes did Kris have left?

19 cupcakes
48. Dan went to lunch at $3: 25$. He returned from lunch at $3: 55$. How many minutes was he at lunch?

30 minutes
49. Miguel walked to his grandfather's house. He left home at 4:55. He arrived at his grandfather's at 5:20. How many minutes did it take Miguel to walk from his home to his grandfather's?

## 25 minutes

50. Ken goes to the store and buys a shirt that costs $\$ 5.99$. He gives the cashier a ten dollar bill. How much change will Ken receive? (Subtraction)

$$
\$ 4.01
$$

Subtracting Integers

## Student Practice Worksheet

Rubric

| Criteria |  |  |  |  |  |
| :--- | :---: | :--- | :--- | :--- | :---: |
|  | 4 | 3 | 2 | 1 | 0 |
| Mechanics | No math errors | No major math <br> errors or serious <br> flaws in reasoning | May be some <br> serious math <br> error or flaws in <br> reasoning | Major math errors <br> or serious flaws <br> in reasoning | Blank <br> answers |

Quiz Grading Rubric:

| Problem | Total points of <br> Correct <br> Answer |  | Problem | Total points of <br> Correct <br> Answer |  |
| :---: | :---: | :--- | :--- | :--- | :--- |
| 1. |  |  | 26. |  |  |
| 2. |  |  | 27. |  |  |
| 3. |  |  | 28. |  |  |
| 4. |  |  | 29. |  |  |
| 5. |  |  | 30. |  |  |
| 6. |  |  | 31. |  |  |
| 7. |  |  | 32. |  |  |
| 8. |  |  | 33. |  |  |
| 9. |  |  | 34. |  |  |
| 10. |  |  | 35. |  |  |
| 11. |  |  | 36. |  |  |
| 12. |  |  | 38. |  |  |
| 13. |  |  | 39. |  |  |
| 14. |  |  | 40. |  |  |
| 15. |  |  | 42. |  |  |
| 16. |  |  | 43. |  |  |
| 17. |  |  | 44. |  |  |
| 18. |  |  | 45. |  |  |
| 19. |  |  | 46. |  |  |
| 20. |  |  |  | 47. |  |
| 21. |  |  | 49. |  |  |
| 22. |  |  |  | 50. |  |
| 23. |  |  |  |  |  |
| 25. |  |  |  |  |  |

Quiz
Name $\qquad$ Date $\qquad$ Grade $\qquad$

Answer the following questions without a calculator. Use the number line for assistance if needed.

## Add or subtract the following integers and expressions:

1. $-31-(-22)$
2. $(-8)+(79)$
3. $11-(-46)$
4. $7-(28)-(-10)$
5. $(-14)+(80)-(-21)$
6. (66) - (68) - (61)
7. $-18 p-25 p$
8. $-16 m-(-38 m)$
9. $6 n-(-8 n)-(-5 n)$

Evaluate each of the following expressions if $\mathbf{a}=\mathbf{- 5}, \mathrm{b}=\mathbf{- 3}$, and $\mathrm{c}=7$ :
10. $a+(-6)$
11. $\mathrm{c}-\mathrm{b}$
12. $\mathrm{b}-(\mathrm{b})$
13. $a+b-c-a$

## Solve the following real world applications to simple addition problems:

14. Francine goes to the grocery store and buys a sack of potatoes for $\$ 15$ dollars, a gallon of milk for $\$ 2.50$, and two cereal boxes for $\$ 8.00$. How much will she receive from the cashier if she pays with a $\$ 50.00$ bill?
15. Arthur sees the thermostat rise 90 degrees from zero, then come down 55 degrees, and then go up 110 degrees, then drop 75 degrees. What is the current temperature in the thermostat?

Quiz
Answer Key
Name $\qquad$ Date $\qquad$ Grade $\qquad$

Answer the following questions without a calculator. Use the number line for assistance if needed.

## Add or subtract the following integers and expressions:

1. $-31-(-22)$
2. $(-8)+(79)$
3. $11-(-46)$
-9
71
57
4. $7-(28)-(-10)$
5. $(-14)+(80)-(-21)$
6. (66) - (68) - (61)
-11
87
-63
7. $-18 p-25 p$
8. $-16 m-(-38 m)$
9. $6 n-(-8 n)-(-5 n)$
-43p
22 m
19

Evaluate each of the following expressions if $a=\mathbf{- 5}, b=-3$, and $c=7$ :
10. $a+(-6)$
11. $\mathrm{c}-\mathrm{b}$
12. $\mathrm{b}-(\mathrm{b})$
13. $a+b-c-a$
$-11$
10
0
-10

## Solve the following real world application to simple addition problems:

14. Francine goes to the grocery store and buys a sack of potatoes for $\$ 15$ dollars, a gallon of milk for $\$ 2.50$, and two cereal boxes for $\$ 8.00$. How much will she receive from the cashier if she pays with a $\$ 50.00$ bill?
15. Arthur sees the thermostat rise 90 degrees from zero, then come down 55 degrees, and then go up 110 degrees, then drop 75 degrees. What is the current temperature in the thermostat?

70 degrees

Subtracting integers

> Quiz

Rubric

| Criteria |  |  |  |  |  |
| :--- | :---: | :--- | :--- | :--- | :---: |
|  |  |  |  |  |  |
| Mechanics | No math errors | No major math <br> errors or serious <br> flaws in reasoning. | May be some <br> serious math <br> error or flaws in <br> reasoning. | Major math errors <br> or serious flaws <br> in reasoning. | Blank <br> answers |

Quiz Grading Rubric:

| Problem | Total points of <br> Correct <br> Answer |  | Problem | Total points of <br> Correct <br> Answer |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. |  |  | 9. |  |  |
| 2. |  |  | 10. |  |  |
| 3. |  |  | 11. |  |  |
| 4. |  |  | 12. |  |  |
| 5. |  |  | 13. |  |  |
| 6. |  |  | 14. |  |  |
| 7. |  |  | 15. |  |  |
| 8. |  |  |  |  |  |

Total: $\qquad$


