Rational Numbers
Rational Numbers and the Absolute Value

Lesson Plan

I. Topic: Rational Numbers and the Absolute Value

II. Goals and Objectives:
A. The students will demonstrate understanding of the absolute value.
B. The students will learn how to interpret the distance from zero as a positive entity.
C. The students will learn how to simplify expressions that have absolute value.
D. The students will distinguish the difference between numbers outside and inside the absolute value sign.

III. Southern Union Mathematics Standards:
1. PA.2.1 Concepts (number sense, algebraic and geometric thinking, measurement, data analysis and probability)
2. PA.2.2 Problem-solving process (explore, plan, solve, verify.)
3. PA.4.1 Simplify expressions using the order of operation.
4. PA.4.2 Identify numbers and relationship among numbers.
5. PA.5.1 Use and evaluate expressions involving variables.
6. AI.4.1 Simplify expressions using the order of operations, including properties of exponents, square roots and absolute value.

IV. Materials:
A. Whiteboard with dry-erase markers (Blackboard with chalk could also be used.)
B. Ruler
C. Pencils
E. Integers and the Absolute Value Worksheets (Practice Worksheet, Quiz Worksheet)
V. Presentation Outline:
   A. Introduction: "Rational Numbers and the Absolute Value"
   B. Key Concepts
   C. Find the absolute value of the given integers
      Examples
   D. Evaluate the expressions with absolute value
      Examples

VI. Presentation:
   A. Presentation Notes
   B. Power Point Presentation

VII. Independent Practice: Rational Numbers and the Absolute Value Worksheet
   A. Class work: #2 - 44 Even
   B. Homework: #s 1 - 45 Odds
   C. Due 2 days from the day assigned. Allow students to complete those questions which they did not complete in class.

VIII. Topic Assessment: Rational Numbers and the Absolute Value Quiz
   A. Answer questions from homework.
   B. 15-question Quiz: 20 – 25 minutes
Rational Numbers

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Rational Numbers and the Absolute Value

Introduction

Vocabulary

Negative Number
A number less than zero

Integers
Numbers includes zero and all positive and negative non-decimal, non-fraction numbers.

Absolute Value
The distance a number is from zero on the number line

Absolute Value
The concept of absolute value has many uses, but for now, you should view the absolute value of a number as that number’s distance from zero.

Let's look at the number line:

| -3 | -2 | -1 | 0 | 1 | 2 | 3 |

The absolute value of x, denoted "| x |" (and which is read as "the absolute value of x"), is the distance of x from zero. This is why absolute value is never negative.

Absolute value only asks "How far?" not "In which direction?"

This means not only that | 3 | = 3, because 3 is three units to the right of zero, but also that | −3 | = 3, because −3 is three units to the left of zero.

Note:
The absolute value notation is bars | |. It is not parentheses ( ) or brackets { }.

It is important to note that the absolute value bars do NOT work in the same way as do parentheses. Whereas − (−3) = +3, this is NOT how it works for absolute value.
In other words:
Simplify $-| -3 |$.

Given $-| -3 |$, I first handle the absolute value part, taking the positive and converting the absolute value bars to parentheses:

$$-| -3 | = -(+3)$$

Now I can take the negative through the parentheses:

$$-| -3 | = -(3) = -3$$

As this illustrates, if you take the negative of an absolute value, you will get a negative number for your answer.

Solve the following examples:

Example I
Fill in the missing space with $<$, $>$, or $=$.

$$| -7.5 | \quad \quad \quad \quad | 7.5 |$$

Since $|-7.5| = 7.5$ and $|7.5| = 7.5$, then the two expressions are equal.

$$| 7.5 | = | 7.5 |$$

Example II
Fill in the missing space with $<$, $>$, or $=$.

$$-4 \quad \quad \quad \quad | -4 |$$

Since $|-4| = 4$, and $-4$ is to the left of $4$ on the number line then $-4 < | -4 |$

Example III
Fill in the missing space with $<$, $>$, or $=$.

$$-8 \quad \quad \quad \quad | -1 |$$

Since $|-1| = 1$ and $1$ is to the right of $-8$, then $-8 < | -1 |$

Example IV
Simplify $| -3 |$.

$$| -3 | = 3$$

Example V
Simplify $| 0 - 5 |$.

$$| 0 - 5 | = |-5| = 5$$
Example VI

Simplify \(| 8 - 3 |\).
\(| 8 - 3 | = | 5 | = 5\)

Example VI

Simplify \(| -2 - 6 |\).
\(| -2 - 6 | = | -8 | = 8\)

Example VII

Simplify \(| 2 + 3(-4) |\).
\(| 2 + 3(-4) | = | 2 - 12 | = | -10 | = 10\)

Example VIII

Simplify \(\neg | (-2)2 |\).
\(\neg | (-2)(2) | = \neg | 4 | = -4\)

Example IX

Simplify \(\neg | -2 |2\)
\(\neg | -2 |2 = \neg (2)(2) = \neg (4) = -4\)

Example X

Simplify \((\neg | -2 |)(2).\)
\((\neg | -2 |)(2) = (-2)(2) = (-2)(2) = 4\)
Evaluate the expressions with absolute value.

Usually absolute value expressions are represented in the following manner $|x + a|$. When evaluating these expressions, we begin by substituting the value of the variables and then solving or simplifying the expression, always keeping in mind that the bars $|$ represent the absolute value and not a parenthesis or a bracket.

Solve the following examples:

Example 1
Evaluate the following absolute value expression: $|2r - 20|$ if $r = 5$

Solution:
- The given expression is: $|2r - 20|$
- Substitute the value $r = 5$ in the given expression: $|2(5) - 20|$
- Simplify this equation using multiplication: $|10 - 20|$
- Subtract the numbers: $|-10|$
  
  The absolute the value of $|-10|$ is 10.

Example 2:
Evaluate the following absolute value expression: $|f^2 - e^2|$ if $e = 13$ and $f = 11$

Solution:
- The given expression is: $|f^2 - e^2|$
- Substitute the value $e = 13$ and $f = 11$: $|11^2 - 13^2|$
- Square the given numbers: $|121 - 169|$
- Subtract the numbers: $|-48|$
  
  The absolute the value of $|-48|$ is 48.

Example 3:
Evaluate the following absolute value expression: $|\sqrt{x} - 10|$ if $x = 100$

Solution:
- The given expression is: $|\sqrt{x} - 10|$
- Substitute the value $x = 100$: $|\sqrt{100} - 10|$
- Take the square root of the value 100: $|10 - 10|$
- Subtract the numbers: $|0|$
  
  The absolute the value of $|0|$ is 0.
Rational Numbers

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Student Practice Worksheet

Name____________________________________________Date______________Grade____________

Answer the following questions about the absolute value of rational numbers.

Find the absolute value of the given rational numbers:

1. $|−2|$  
2. $|18|$  
3. $|2.5|$  

4. $\left|\frac{-5}{6}\right|$  
5. $|−38|$  
6. $|10|$  

7. $|97|$  
8. $|−61|$  
9. $|3.9|$  

10. $|−6.8|$  
11. $\left|\frac{-23}{56}\right|$  
12. $\left|\frac{35}{80}\right|$  

13. $|−4|$  
14. $|3|$  
15. $\left|\frac{19}{3}\right|$  

16. $−|−109|$  
17. $|1.48|$  
18. $−|103.25|$  

19. $−|−13.64|$  
20. $|−6.7|$  
21. $\left|\frac{2}{3}\right|$  

22. $\left|\frac{-7}{11}\right|$  
23. $−\left|\frac{-7}{11}\right|$  
24. $−\left|\frac{7}{11}\right|$  

25. $|471.378|$  
26. $|−23.76|$  
27. $−|23.76|$
28. $\left| -\frac{16}{2} \right|$

29. $|72|$

30. $-|-72|$

Evaluate each expression if $a = 6$, $b = \frac{2}{3}$, $c = \frac{5}{4}$, $x = 12$, $y = 3.2$, and $z = -5$.

31. $57 - |x + 34|$

32. $19 + |21 - y|$

33. $|z| - 0.26$

34. $48 + |x - 5|$

35. $|43 - 4a| + 51$

36. $6.5 - |8.4 - y|$

37. $\left( b + \frac{1}{2} \right) - \left| -\frac{5}{6} \right|$

38. $25 + |17 + x|$

39. $|z| + 13 - 4$

40. $7.4 + |y - 2.6|$

41. $|c - 1| + \frac{2}{5}$

42. $|17 - a| + 23$

43. $28 - 13 + |z|$

44. $\frac{1}{6} + \left| b - \frac{7}{12} \right|$

45. $|-c| + \left( 2 + \frac{1}{2} \right)$
Rational Numbers and the Absolute Value

Student Practice Worksheet

Answer Key

Name______________________________________________ Date______________ Grade_________

Answer the following questions about the absolute value of rational numbers.

Find the absolute value of the given rational numbers:

1. $|{-2}|$  
   2

2. $|18|$  
   18

3. $|2.5|$  
   2.5

4. $|rac{-5}{6}|$  
   $\frac{5}{6}$

5. $|{-38}|$  
   38

6. $|10|$  
   10

7. $|97|$  
   97

8. $|{-61}|$  
   61

9. $|3.9|$  
   3.9

10. $|{-6.8}|$  
    6.8

11. $|{-\frac{23}{56}}|$  
    $\frac{23}{56}$

12. $|\frac{35}{80}|$  
    $\frac{35}{80}$

13. $|{-4}|$  
    4

14. $|3|$  
    3

15. $|\frac{19}{3}|$  
    $\frac{19}{3}$

16. $|{-109}|$  
    -109

17. $|1.48|$  
    1.48

18. $|{-103.25}|$  
    -103.25

19. $|{-13.64}|$  
    -13.64

20. $|{-6.7}|$  
    6.7

21. $|\frac{2}{3}|$  
    $\frac{2}{3}$

22. $|{-\frac{7}{11}}|$  
    $\frac{7}{11}$

23. $|{-\frac{7}{11}}|$  
    $-\frac{7}{11}$

24. $|\frac{7}{11}|$  
    $\frac{7}{11}$

25. $|471.378|$  
    471.378

26. $|{-23.76}|$  
    23.76

27. $|{-23.76}|$  
    23.76
28. \[ \left| -\frac{16}{2} \right| \]
29. \[ |72| \]
30. \[ -|-72| \]

\[ 8 \quad 72 \quad -72 \]

Evaluate each expression if \( a = 6, b = \frac{2}{3}, c = \frac{5}{4}, x = 12, y = 3.2, \) and \( z = -5. \)

31. \[ 57 - |x + 34| \]
32. \[ 19 + |21 - y| \]
33. \[ |z| - 0.26 \]

\[ 11 \quad 36.8 \quad 4.74 \]

34. \[ 48 + |x - 5| \]
35. \[ |43 - 4a| + 51 \]
36. \[ 6.5 - |8.4 - y| \]

\[ 55 \quad 70 \quad 1.3 \]

37. \[ \left( b + \frac{1}{2} \right) - \left| -\frac{5}{6} \right| \]
38. \[ 25 + |17 + x| \]
39. \[ |z| + 13 - 4 \]

\[ \frac{1}{3} \quad 54 \quad 14 \]

40. \[ 7.4 + |y - 2.6| \]
41. \[ |c - 1| + \frac{2}{5} \]
42. \[ |17 - a| + 23 \]

\[ 8 \quad \frac{13}{20} \quad 34 \]

43. \[ 28 - 13 + |z| \]
44. \[ \frac{1}{6} + \left| b - \frac{7}{12} \right| \]
45. \[ |c| + \left( 2 + \frac{1}{2} \right) \]

\[ 20 \quad \frac{1}{4} \quad \frac{15}{4} \]
### Integers and the Absolute Value

#### Student Practice Worksheet

**Rubric**

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<tr>
<th>Criteria</th>
<th>4</th>
<th>3</th>
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**Quiz Grading Rubric:**

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**Total Points = _________**
Integers and the Absolute Value

Quiz

Name_________________________ Date_________ Grade________

Answer the following questions about the absolute value of rational numbers.

Find the absolute value of the given rational numbers:

1. $|{-26}|$ 
2. $|100|$ 
3. $|{-0.35}|$

4. $\left|\frac{-28}{53}\right|$ 
5. $-|7.8|$ 
6. $|1|$

Evaluate each expression if $a = 4$, $b = \frac{3}{5}$, $c = \frac{3}{2}$, $x = 14$, $y = 2.4$, and $x = -3$:

7. $41 - 16 - |z|$ 
8. $\left(b - \frac{1}{5}\right) + \left|-\frac{3}{10}\right|$ 
9. $|3a + 20| - 15$

10. $\frac{2}{15} + \left|b - \frac{2}{5}\right|$ 
11. $|2x + 4| - 6$ 
12. $|c - 1| - \frac{1}{3}$

13. $2.5 - |3.8 - y|$ 
14. $|-c| - \frac{3}{4}$ 
15. $7.4 + |y - 2.6|$
Integers and the Absolute Value

Quiz

Answer Key

Name________________________________________Date________________Grade_________

Answer the following questions about the absolute value of rational numbers.

Find the absolute value of the given rational numbers:

1. \( |−26| \)
2. \( |100| \)
3. \( |−0.35| \)

\[26 \quad 100 \quad 0.35\]

4. \( \left| \frac{−28}{53} \right| \)
5. \( −|7.8| \)
6. \( |1| \)

\[\frac{28}{53} \quad -7.8 \quad 1\]

Evaluate each expression if \( a = 4 \), \( b = \frac{3}{5} \), \( c = \frac{3}{2} \), \( x = 14 \), \( y = 2.4 \), and \( x = -3 \):

7. \( 41 − 16 − |z| \)
8. \( \left( b − \frac{1}{5} \right) + \left| −\frac{3}{10} \right| \)
9. \( |3a + 20| − 15 \)

\[22 \quad \frac{7}{10} \quad 17\]

10. \( \frac{2}{15} + \left| b − \frac{2}{5} \right| \)
11. \( |2x + 4| − 6 \)
12. \( |c − 1| − \frac{1}{3} \)

\[\frac{1}{3} \quad 26 \quad \frac{1}{6}\]

13. \( 2.5 − |3.8 − y| \)
14. \( |−c| − \frac{3}{4} \)
15. \( 7.4 + |y − 2.6| \)

\[1.1 \quad \frac{3}{4} \quad 7.6\]
Integers and the Absolute Value

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