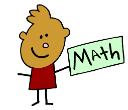


Math Musts:

The Fundamentals of Higher Mathematics

The basics of Math
explained for grades 7 – 12,
laying the foundation
for higher math.

Table of Contents

Introduction

To succeed in higher levels of mathematics, every math student should have a solid grasp of the basic concepts which form the foundation for higher math. Knowing how math "works" and why it "works" in certain ways can lead to improved interest, a better appreciation for the beauty in math, or perhaps less math anxiety and intimidation. Once students begin to understand the basics, I've heard students say, "This is actually fun," "Now it makes sense," or "This isn't so bad once you get the hang of it." Comments like these prove that everyone can learn math, if taught properly.

This unit attempts to explain the fundamental principles that govern higher math courses such as Advanced Algebra, Geometry, Pre-Calculus, Probability & Statistics, and Calculus. Not every principle is covered, but the majority of them are. With this knowledge students will have a solid grasp of parts of math that answer the "Why?" questions — "Why do we do it that way?" "Why won't this method work instead?" "Will this way always work?"

However, the inevitable math student question, "Why do I have to learn this?" is still up to you to answer. But here's some help!

Analogy: When a house is built, builders begin by digging a hole in the ground, and eventually pour a concrete foundation. A lot of the material covered in basic algebra courses, all the way through Pre-Calculus, is similar to "laying the foundation" for higher math courses. Many times there is no direction application to a real-world problem. However, the concepts learned could someday be used to solve much more difficult, real-world problems.

Also, the study of mathematics is like doing mental gymnastics. It keeps your mind agile, strong, and adaptable. Have fun with it, and see if you can appreciate the beauty in it.

"Do not worry about your difficulties in mathematics, I assure you that mine are greater." – Albert Einstein

Math Must 1: Sets of Numbers

The word "set" has lots of different meanings. In the context of math, the dictionary defines a set as a "number of things combining to form a whole; a complete assortment." And that's exactly what this lesson is about—defining an assortment of numbers.

In math, we will use braces to denote a set: { }. Each number inside should be separated by a comma. Ok, here goes...

Natural Numbers

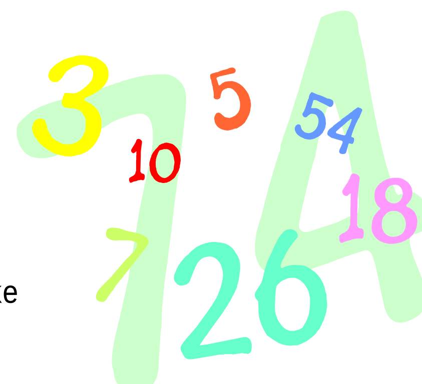
The most basic set of numbers is the set of natural numbers (naturals). Sometimes naturals are referred to as counting numbers. Get this—when you *count*, you *naturally* start with one, two, three, etc. (Neat way to remember the name of this set, huh?)

Natural Numbers = { 1, 2, 3, 4, 5, 6, ... } *We can't write them all!*

Whole Numbers

This set is made up of all the numbers in the naturals plus one more number – zero. So whole numbers include the "hole" (hold up your hands like a donut). (Another neat way to remember its name, huh?)

Whole Numbers = { 0, 1, 2, 3, 4, 5, 6, ... }



Integers

One of the most popular words in math is integer. Integers are made up of all the whole numbers and their opposites (negatives). Just remember that all numbers have an opposite except for zero. Positive zero or negative zero just doesn't make sense.

Integers = { ..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ... }

Rationals

Here's where things start to get a bit complicated. Perhaps the simplest way to describe this set is, it includes all numbers that can be written as a fraction. That means that this set includes the integers, because each integer can be written as a fraction. How's that, you ask? Simple. Just take any integer, say 3, for example, and place it over the number one, like this: 3/1. Now you've written the number 3 as a fraction. It even works with zero!

Of course rationals also include positive and negative numbers like $\frac{1}{2}, \frac{5}{8}, -\frac{9}{4}$, or $-7\frac{3}{16}$. One last thing about rationals—they also include certain decimals. Specifically, if a decimal stops, like 0.45 or 12.56983, these decimals can be written as a fraction. Also, if a decimal continues on and on forever, but has a repeatable pattern, like 0.333333... or 0.123123123123..., then it too can be written as a fraction.

Let's try something. Got a calculator nearby? You're going to need it.

The trick is dividing by 9. Take the decimal number 0.333333... Since the single digit three repeats over and over, to get the decimal on your calculator, divide three by nine ($3 \div 9 =$). But in the second case of 0.123123123..., the pattern has three digits repeating. So on your calculator, divide 123 by 999 ($123 \div 999 =$). Neat! This is explained further in Math Must 6: Decimals and Percents.

The pattern must be repeatable. Here's a pattern: 0.12122122212222..., but how many nines would you divide by? This decimal is not a rational number.

Rationals = { any number that can be written as a fraction }

This calculator is just a few billion digits short of pi.

<u>Irrationals</u>

Well, if rationals are numbers that <u>can</u> be written as a fraction, guess what Irrationals are? That's correct! Irrationals are numbers that <u>cannot</u> be written as a fraction. Irrationals are decimals that go on and on forever, but do not have a repeatable pattern.

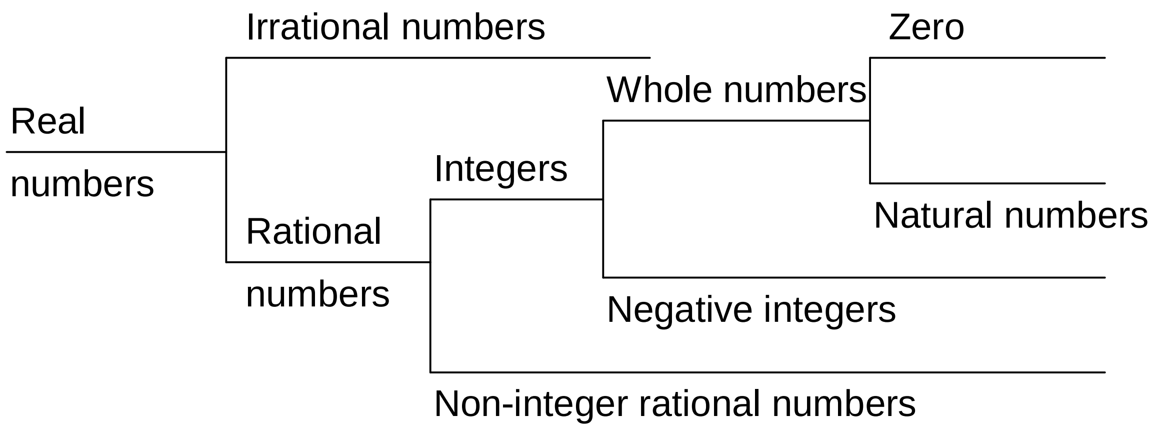
Arguably, the most popular irrational number is pi (π). Pi is a decimal that goes on forever, without a detectable pattern (3.14592654...). Can you believe that people, with the help of modern day computers, have figured out pi to over 2 billion decimal places, still without a detectable pattern?

Another common irrational number is the radical ($\sqrt{\ }$). Find the square root of 2 or the square root of 3 using a calculator ($\sqrt{2} \approx 1.414213562$ and $\sqrt{3} \approx 1.732050808$). But watch out! Not every radical is irrational. Find the square root of 4 using a calculator ($\sqrt{4} = 2$). As you already learned, the number 2 is rational. It can be written as a fraction: $\frac{2}{1}$. You will find more information on radicals at the end of this lesson.

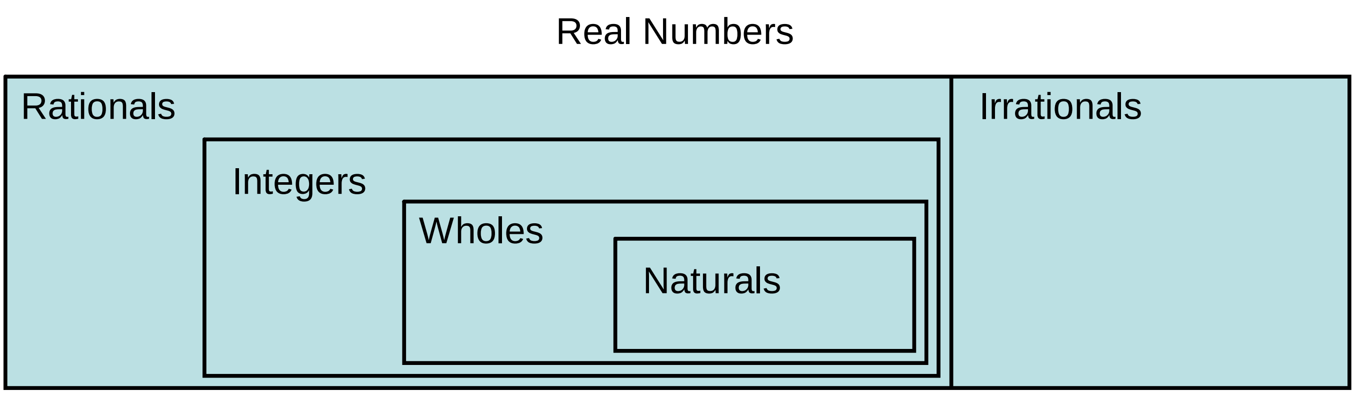
Irrationals = { any number that cannot be written as a fraction } *or* = { non-Rational numbers }

<u>Real Numbers</u>

Finally, the last set of numbers in this lesson is the set of Real Numbers. (Does that mean there must also be a set of "Fake Numbers"? Yep, but that set is not covered in this unit.) Real numbers include all the sets of numbers you've learned so far—Naturals, Wholes, Integers, Rationals, and Irrationals. Every number discussed so far is Real. Here is the relationship:

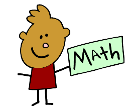


Here's another way to view the relationship between the sets of numbers:



Yes, math is very logical, well planned out, and thought out. It has, for the most part, a nice hierarchical structure to it. Almost as if it were designed and created from a Master Planner. Hmmm…

Sounds like our Creator God at work again! Can you appreciate its beauty?



Let's look at this example problem:

Place each given number in each set of numbers where it belongs.

$$\pi,\ 3,\ -\frac{2}{5},\ 0,\ 1.725,\ \sqrt{10},\ 0.\overline{45},\ -8,\ \frac{29}{5},\ 1.8,\ \sqrt{7},\ 0.\overline{3},\ -4,\ 2,\ 10$$

Reals $\pi,\ 3,\ -\frac{2}{5},\ 0,\ 1.725,\ \sqrt{10},\ 0.\overline{45},\ -8,\ \frac{29}{5},\ 1.8,\ \sqrt{7},\ 0.\overline{3},\ -4,\ 2,\ 10$

Rationals $3,\ -\frac{2}{5},\ 0,\ 1.725,\ 0.\overline{45},\ -8,\ \frac{29}{5},\ 1.8,\ 0.\overline{3},\ -4,\ 2,\ 10$

Integers $3,\ 0,\ -8,\ -4,\ 2,\ 10$

Wholes $3,\ 0,\ 2,\ 10$

Naturals $3,\ 2,\ 10$

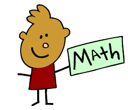
Irrationals $\pi,\ \sqrt{10},\ \sqrt{7}$

Radicals

As was previously mentioned, another group of irrational numbers is square roots (radicals). Most square roots are irrational numbers. The square roots that are not irrational have a square number inside. For example, the square root of 4 equals 2, written this way: $\sqrt{4} = 2$. Notice the answer, 2, is a rational number. So the square root of 4 is also rational. That's because the number 4 is a perfect square (2-squared = 4). Students are asked to find all the perfect square numbers between 1 and 10 on page 3 of the practice problems following this lesson.

Now that you know what a perfect square number is, let's look at the rest of the square roots that are irrational. For example, $\sqrt{57}$ is irrational, since 57 is not a perfect square.

Notice though, that 57 is about half-way between two perfect squares, 49 and 64. So, since $\sqrt{49} = 7$ and $\sqrt{64} = 8$, then wouldn't you expect that $\sqrt{57}$ is equal to some number between 7 and 8? Turns out it is approximately 7.5, right where we'd expect it to be. Verify this on your calculator.



Similarly, $\sqrt{8} \approx 2.8$ since 8 is between the square numbers 4 and 9. In fact 8 is closer to 9 than it is closer to 4. So, the approximate answer of 2.8 makes sense. Since $\sqrt{8}$ should be closer to $\sqrt{9} = 3$ than to $\sqrt{4} = 2$.

One more time, but let's think about the problem a bit differently. We've already established that $\sqrt{4} = 2$ and $\sqrt{9} = 3$. What are the integers between 4 and 9? Answer: 5, 6, 7, and 8. Since there are a total of four integers between the perfect squares of 4 and 9, let's add one more to that number (you'll see why we're doing this in a little while), we get 5. Now, think about how to "break up" or "separate," into 5 equal parts, the interval between 2 and 3. Why between 2 and 3? Because they are the answers to the square roots 4 and 9, respectively. To break this interval up into 5 equal parts, you could do it like this:

$$2.0 \underbrace{\quad - \quad}_{\text{one}} 2.2 \underbrace{\quad - \quad}_{\text{two}} 2.4 \underbrace{\quad - \quad}_{\text{three}} 2.6 \underbrace{\quad - \quad}_{\text{four}} 2.8 \underbrace{\quad - \quad}_{\text{five}} 3.0$$

Now watch this!

2.0	–	2.2	–	2.4	–	2.6	–	2.8	–	3.0
↑		↑		↑		↑		↑		↑
$\sqrt{4}$		$\sqrt{5}$		$\sqrt{6}$		$\sqrt{7}$		$\sqrt{8}$		$\sqrt{9}$

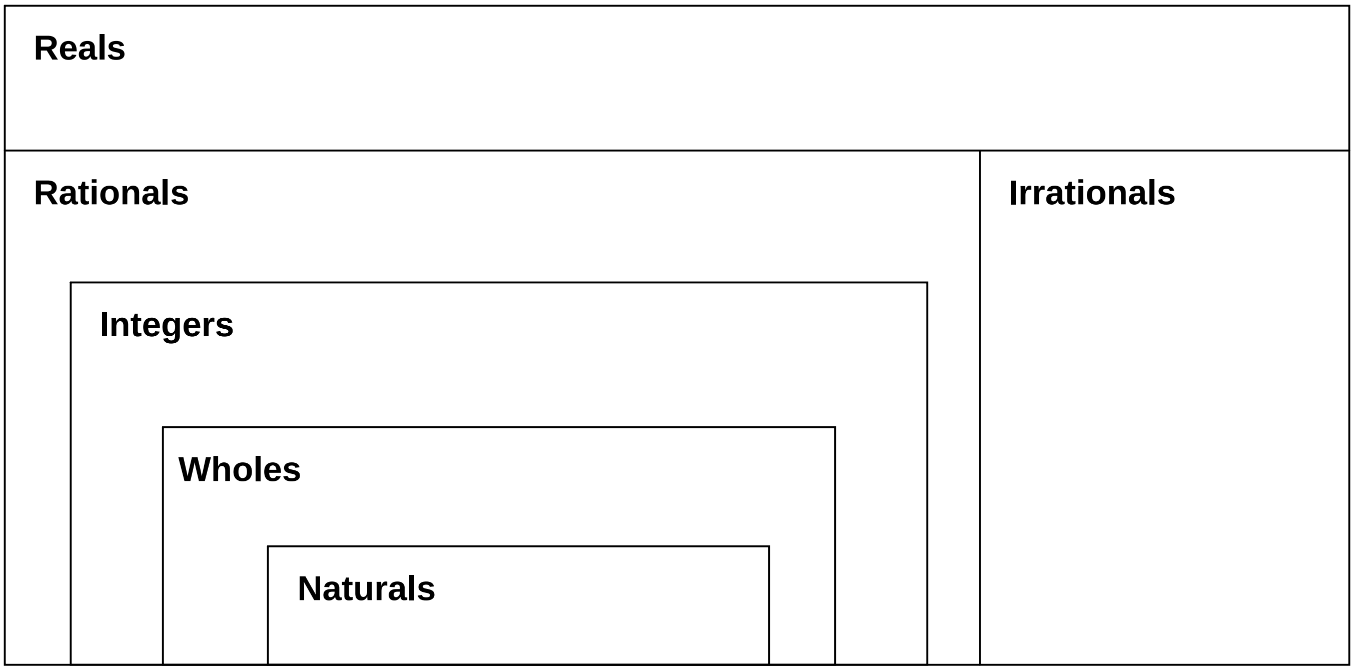
Look, $\sqrt{8} \approx 2.8$
It matches up!

Name ____ Date ________________

Practice: Sets of Numbers

1. Place the following numbers into each set they belong in.

$$-5.3,\ \ -5,\ \ -\sqrt{3},\ \ -1,\ \ -\frac{1}{9},\ \ 0,\ \ 1.2,\ \ 4,\ \ \sqrt{12}$$



2. Complete the following table:

Number	Whole	Integer	Rational	Irrational	Real
10	yes			no	Yes
-2		yes		no	
$1/3$	no				
$-\sqrt{3}$					
2.5			yes		
$\frac{\pi}{2}$	no				
$\sqrt{100}$		yes			

Name ____ Date ________________

Practice: Sets of Numbers Page 2

1. Arguably the most popular irrational number is pi (π). Use a book, encyclopedia, or the internet to determine π to at least 20 decimal places.

$\pi \approx$ ____

2. A **mnemonic** device is a scheme where you can recall facts by memorizing something completely unrelated to facts. One way of learning the first few digits of the decimal for π is to memorize a sentence (or several sentences) and count the letters in each word of the sentence. For example, "See, I know a digit," will give the first 5 digits of π: "See" has 3 letters, "I" has 1 letter, "know" has 4 letters, "a" has 1 letter, and "digit" has 5 letters. So the first five digits are 3.1415.

Verify that the following mnemonic devices work. Use the decimal for π you wrote in Exercise 1 above.

a) "May I have a large container of donuts?" π = ____________________________________

b) "See, I have a rhyme assisting my feeble brain, its tasks ofttimes resisting."

π = ____

c) "How I need a break, unwinding of course, after the heavy lectures involving quantum mechanics."

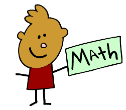
π = ____

3. Make up your own mnemonic device to obtain the first eight digits of π.

____

____

____



Name ______________________________ Date ______________

Practice: Sets of Numbers

Find the following squares.

1^2 = ________ 6^2 = ________

2^2 = ________ 7^2 = ________

3^2 = ________ 8^2 = ________

4^2 = ________ 9^2 = ________

5^2 = ________ 10^2 = ________

Without using a calculator, approximate the following radicals to one decimal place.

1. $\sqrt{5} \approx$ ____________ 2. $\sqrt{10} \approx$ ____________

3. $\sqrt{15} \approx$ ____________ 4. $\sqrt{50} \approx$ ____________

5. $\sqrt{40} \approx$ ____________ 6. $\sqrt{70} \approx$ ____________

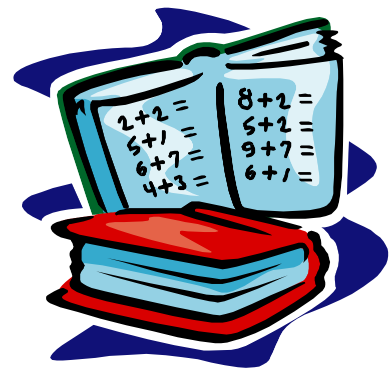
7. $\sqrt{82} \approx$ ____________ 8. $\sqrt{33} \approx$ ____________

9. $\sqrt{93} \approx$ ____________ 10. $\sqrt{120} \approx$ ____________

Math Must 2: Properties of Real Numbers

The following five properties are, for the most part, pretty straight forward and simple. However, their use in math is far-reaching, well into Calculus.

<u>Commutative Property</u>
Every day we attend school, work, or church, we find ourselves commuting. The word *commute* means to go back and forth. Perhaps your commute to school today was by car, bus, bicycle, or walking. Well, numbers can commute too. This is done through two math operations—addition and multiplication. (Note: The commutative property does not apply to subtraction and division. Why not?)



Let's suppose that the letters (variables) *a* and *b* represent real numbers. Then, formally the commutative property looks like this:

$a + b = b + a$	Addition
$a \times b = b \times a$	Multiplication

Notice that on the left-side of the equal sign the number *a* is written before the number *b*. And on the right-side of the equal sign the number *b* is written before the number *a*. So, as long as you are adding or multiplying, the order of the numbers does not matter.

$6 + 3 = 9$	is the same as	$3 + 6 = 9$
$6 \times 3 = 18$	is the same as	$3 \times 6 = 18$

The numbers were *commuted*.

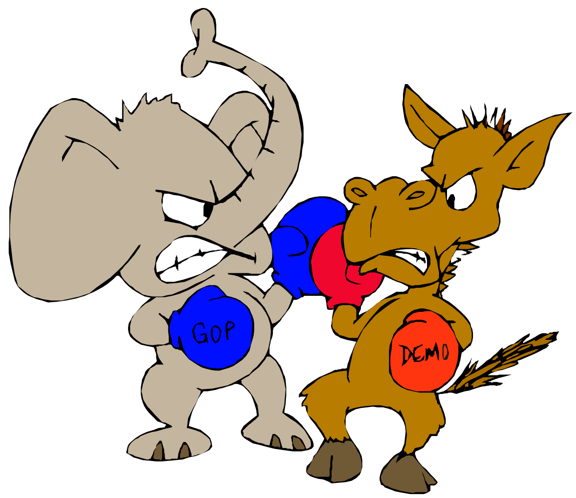
Demonstration: Have two students come to the front of the room and stand next to each other. Sum up their heights (in inches). Now have them switch places. If you sum up their heights again, does the total change? No, it shouldn't matter.

<u>Associative Property</u>
This property, like the commutative property, only applies to addition and multiplication. However, unlike the commutative property, the associative property does not move numbers around, but rather it groups numbers together using parentheses. Because of the order of operations in math, whatever is inside of parentheses must be done first. So, by using grouping symbols like parentheses, the associative property forces you to work with a group of numbers in order.

Let's suppose that the letters (variables) *a, b*, and *c* represent real numbers. Then, formally the associative property looks like this:

$(a + b) + c = a + (b + c)$	Addition
$(a \times b) \times c = a \times (b \times c)$	Multiplication

Try this demonstration: Have three students come to the front of the room and stand next to each other in a line. Have the student on the far left side and the student in the middle look at each other. They are grouped together, or are associating with one another. Sum up their heights (in inches). Now add the sum of their heights with the student to the far right. Next, without switching places, have the student on the far right side and the student in the middle look at each other. This time, they are grouped together, or are associating with one another. Sum up their heights. Now add the sum of their heights with the student to the far left. Does the total change? No, it shouldn't matter.



Although it isn't a math lesson, with the three students up front, it would be a good time to demonstrate political terms—"Liberal" (left), "Conservative" (right), and "Independent" (in the middle or non-partisan). Ok, back to math.

Identity Property

Have you heard of people who've had an identity crisis? They don't know who they are. Perhaps they are trying to "find themselves." Well, this property is similar. By looking in a mirror you see yourself—you don't have to go far to find yourself. That's how the identity property works, like a mirror. Formerly,

$a + 0 = a$	Additive Identity
$a \times 1 = a$	Multiplicative Identity

Notice that in both cases you started with *a* and ended up with *a*. So when adding, zero (0) is the additive identity—it gives you back the same number you started with. And when multiplying, one (1) is the multiplicative identity—it gives you back the same number you started with.

This property seems very simplistic, and to a certain extent, it is. However, it is very powerful, mathematically speaking.

Inverse Property

When you read or hear the word *inverse* in math, think opposite. When adding, the inverse of a number is its opposite (opposite sign). When multiplying, the inverse of a number is its reciprocal (a fancy word for "flip it").

$a + (-a) = 0$	Additive Inverse
$a \times \frac{1}{a} = 1$	Multiplicative Inverse

Four ÷ zero? I'm smart but not that smart!

Watch out! There's one caveat for this property—the number zero (0). Zero does not have an additive inverse. There's no such number as negative zero (–0). Zero also does not have a multiplicative inverse. The fraction $\frac{0}{4}$ is a real number. But if you flip it, $\frac{4}{0}$ is not a number. True! Try it on your calculator (but be warned, it may disintegrate or start to smoke…ok, not really): $4 \div 0 = ?$ What answer did you get? Probably an "Error" message.

Distributive Property

This last property is arguably the most important of them all. It is used throughout high school math, college math, and beyond. Just like the newspaper delivery guy might drive down your street early in the morning, throwing the Sunday newspaper out onto homes' front steps (to those houses who've subscribed of course), this property does something similar.

Check out the following example:

$2(5 + 4)$	
$2(9)$	One way to solve it, is to start in the parentheses, by adding 5 and 4.
18	Then, multiply 2 and 9, which is 18.

$2(5 + 4)$	Another way to do this problem is by using the distributive property. Start by distributing the 2 in front of the parentheses to each term inside.
$2 \times 5 + 2 \times 4$	This is done using multiplication.
$10 + 8$	Now follow the order of operations (multiply before adding).
18	Adding 10 and 8 is 18. Same answer as before.

Formally, the distributive property is:

$$a(b + c) = a \times b + a \times c$$

Name ______________________________________ Date _______________

Practice: Properties of Real Numbers Page 1

Identify the property illustrated by each of the following statements. Choose from the following list:

- Commutative Property
- Associative Property
- Identity Property
- Inverse Property
- Distributive Property

1. ___________________________________ $8 + 0 = 8$

2. ___________________________________ $5 + (6 + 8) = (5 + 6) + 8$

3. ___________________________________ $-4\left(-\frac{1}{4}\right) = 1$

4. ___________________________________ $4 + (3 + 9) = 4 + (9 + 3)$

5. ___________________________________ $5\,(x + y) = 5x + 5y$

6. ___________________________________ $-7 + 7 = 0$

7. ___________________________________ $(3 \bullet 5) \bullet 4 = 4 \bullet (3 \bullet 5)$

8. ___________________________________ $\frac{1}{12} \bullet 12 = 1$

9. ___________________________________ $9 \bullet 6 + 9 \bullet 8 = 9\,(6 + 8)$

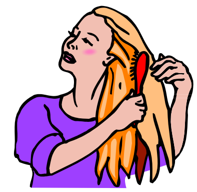
10. ___________________________________ $-50 \bullet 1 = -50$

Name ____ Date ________________

Practice: Properties of Real Numbers Page 2

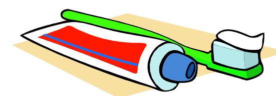
Many everyday activities are commutative; that is, the order in which they occur does not affect the outcome. For example, “putting on your shirt” and “putting on your pants” are commutative operations. Decide whether the given activities are commutative (yes or no):

1. putting on your shoes; putting on your socks ____________



2. getting dressed; taking a shower ____________

3. brushing your hair; brushing your teeth ____________



Create your own commutative operations:

4. ____

Create your own non-commutative operations:

5. ____

Many everyday occurrences can be thought of as operations that have opposites or inverses. For example, the inverse operation for “going to sleep” is “waking up.” For each of the given activities, specify its inverse activity.

6. cleaning up your room; ____

7. earning money; ____

8. increasing the volume on your iPod; ________________________________

Create your own inverse operations:

9. ____

Name ____ Date ________________
Practice: Properties of Real Numbers Page 3

The following conversation actually took place between a math teacher and his son, Jack, when Jack was four years old.

Daddy: "Jack, what is 3 + 0?"
Jack: "3"
Daddy: "Jack, what is 4 + 0?"
Jack: "4…and Daddy, *string* plus zero equals *string*!"

1. What property of real numbers did Jack recognize? ___________________________________

The phrase *defective merchandise counter* is an example of a phrase that can have different meanings depending upon how the words are grouped (think of the associative property). For example, *(defective merchandise) counter* is a location at which we would return an item that does not work, while *defective (merchandise counter)* could be a broken place where items are bought and sold or it could be a device that counts merchandise that isn't working properly. Either way, its meaning could vary, depending on how the words are associated.

For each of the following phrases, explain the two different meanings based upon grouping.

2. woman fearing husband

Meaning #1: ____

Meaning #2: ____

3. man biting dog

Meaning #1: ____

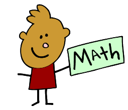
Meaning #2: ____

4. This time, create your own phrase with dual meaning, then explain each meaning.

Your phrase: ____

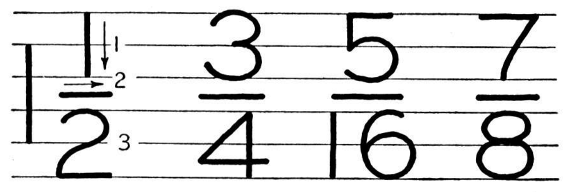
Meaning #1: ____

Meaning #2: ____



Math Must 3: Rational Numbers (Part 1)

If you've already read through Math Must 1: Sets of Numbers, then you know what a rational number is. Recall that rational numbers are numbers that can be written as fractions. So when you read or hear "rational," think "fractional" (not sure that's a real word). But don't be nervous, fractions are easier to deal with than you might think.



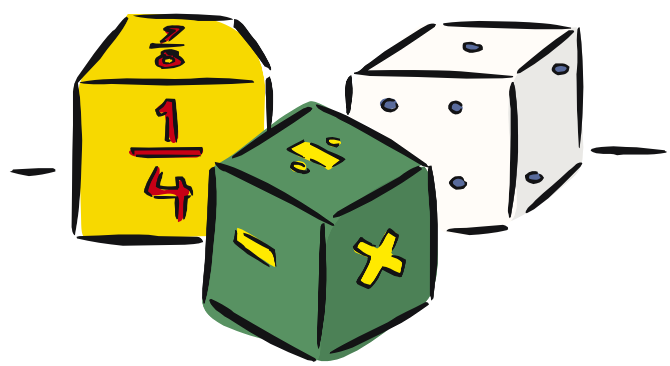
One thought before you start: in higher math mixed fractions are cumbersome to deal with. So I encourage you to leave fractions, if necessary, in improper form (where the numerator—top number—is larger than the denominator—bottom number).

Traditionally, when a math concept is taught, addition and subtraction are shown first, followed by multiplication and division. However, multiplying and dividing fractions are easier to handle than adding and subtracting. So let's begin with multiplication and division first.

<u>Multiplication of Fractions</u>
The rule for multiplying two fractions is extremely simple – multiply straight across. For example:

$$\frac{3}{4} \cdot \frac{7}{10} = \frac{3 \cdot 7}{4 \cdot 10} = \frac{21}{40}$$

Wasn't that easy?



<u>Division of Fractions</u>
The rule for dividing fractions is almost as easy as multiplication. Turns out you change division problems into multiplication problems by doing the following:
1) change the division symbol (÷) into multiplication (•),
2) then flip the second fraction (reciprocal).
Now go back to multiplying. For example:

$$\frac{1}{3} \div \frac{2}{5} = \frac{1}{3} \cdot \frac{5}{2} = \frac{1 \cdot 5}{3 \cdot 2} = \frac{5}{6}$$

Not too bad, huh? Remember, multiply and flip the second fraction only!

Addition / Subtraction of Fractions

First of all, keep this in mind: **to add or subtract fractions, all fractions must have the same denominators**. Sometimes when adding fractions, the denominators are already the same. If that's the case, then adding them is easy. Simply add the numerators (tops) and keep the same denominator (don't add them). Like this:

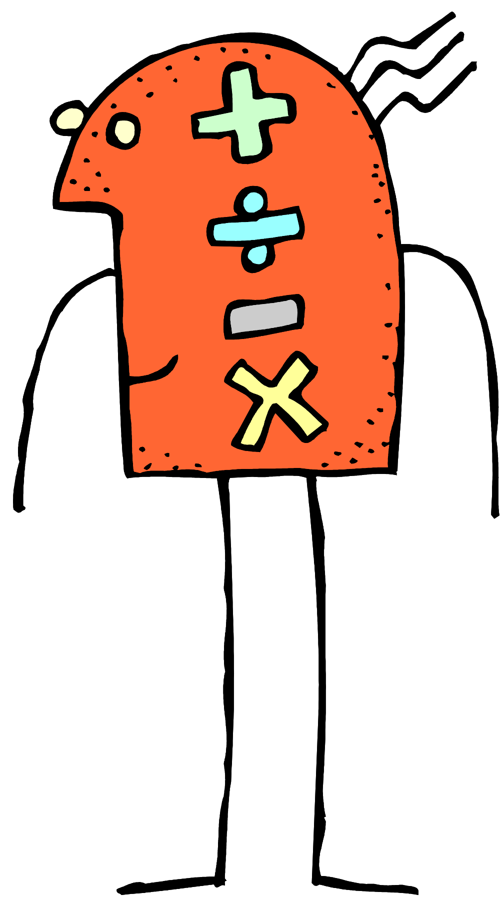
$$\frac{4}{3}+\frac{1}{3}=\frac{4+1}{3}=\frac{5}{3} \qquad \text{Same for subtraction...} \qquad \frac{4}{3}-\frac{2}{3}=\frac{4-2}{3}=\frac{2}{3}$$

Sometimes, though, when adding fractions, the denominators are not the same. That's when things get a bit more complicated. Keep in mind, to add or subtract fractions, all fractions must have the same denominators. So, if the denominators are not the same, then let's make them the same. Here's how.

Recall a couple of things:

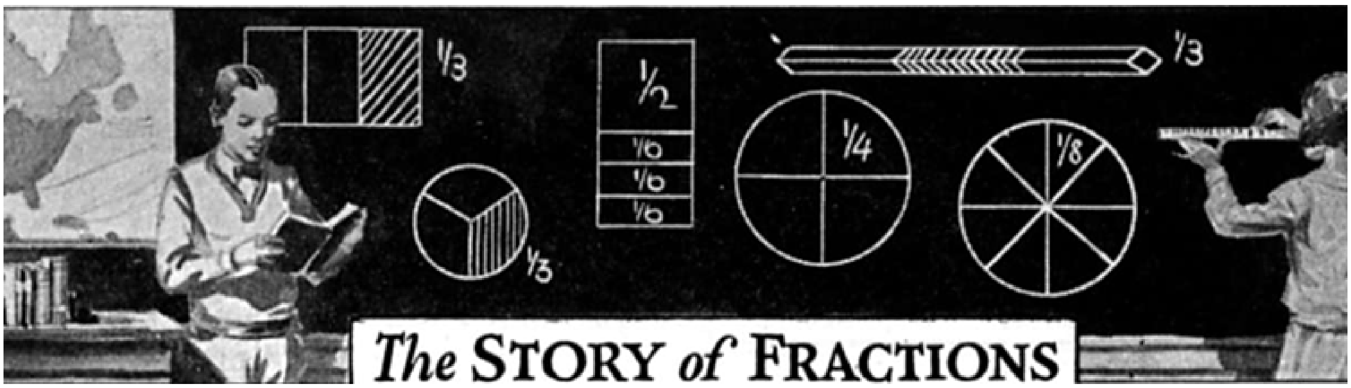
- any number divided by itself equals one (inverse property)
- any number times one equals itself (identity property)

Let's apply those properties to fractions. Any fraction can be multiplied by the number one. According to the identity property, that shouldn't change it. It may look slightly different, but it's the same fraction. So if you had to add these two fractions, $\frac{1}{4}+\frac{3}{8}$ where the denominators are not the same, you'd have to change something so that they are the same. Why not multiply the first fraction, $\frac{1}{4}$, by the number one. Like this: $1 \bullet \frac{1}{4}$. This is applying the identity property. It does not change the fraction. But, a fancy way to write the number one, is to write it as 2 divided by itself: $\frac{2}{2} \bullet \frac{1}{4}$. This is applying the inverse property. Pretty neat huh? And, what's the rule for multiplying fractions? Multiply straight across, giving $\frac{2}{8}$. So this new fraction, $\frac{2}{8}$, is the same as the one you started with, $\frac{1}{4}$. It just looks a little different.



Now the two fractions we're adding are: $\frac{2}{8}+\frac{3}{8}$. And since the denominators are the same, you simply add the numerators together, keeping the same denominators.

Final answer: $\frac{2}{8}+\frac{3}{8}=\frac{2+3}{8}=\frac{5}{8}$. The same trick works for subtracting fractions. It also works when both original fractions need to be changed so that you have a common denominator. Oh...the power of the number one!



Fraction Facts:

The numerator (top) and denominator (bottom) of a fraction may be separated by a slanting line called a *solidus*.

For example, $^{3}/_{4}$. *solidus*

Or numerators and denominators may be written above and below a horizontal line called a *vinculum*.

For example, $\frac{3}{4}$. *vinculum*

Your task—use these words in front of your family and friends to really impress them! Or better yet, give your next child or grandchild the name "Solidus." Good luck with that!

Name ____ Date ________________

Practice: Rational Numbers (Part 1) Page 1

Multiply or divide. If necessary, a whole number can be written as a rational number by putting it over one.

1. $\frac{4}{5} \bullet \frac{6}{7}$

2. $\frac{5}{9} \bullet \frac{10}{7}$

3. $\frac{7}{3} \bullet 7$

4. $\frac{3}{10} \div \frac{2}{7}$

5. $\frac{5}{4} \div \frac{3}{8}$

6. $\frac{3}{4} \div 5$

Add or subtract.

7. $\frac{6}{12} + \frac{1}{12}$

8. $\frac{4}{5} - \frac{1}{5}$

9. $\frac{5}{9} + \frac{1}{3}$

10. $\frac{1}{5} + \frac{4}{15}$

11. $\frac{3}{4} + \frac{2}{3}$ *(Hint: make the denominators 12)*

12. $\frac{7}{6} - \frac{1}{5}$ *(Hint: make the denominators 30)*

Name ______________________________________ Date ______________

Practice: Rational Numbers (Part 1) Page 2

1. The Parthenon in Athens has measurements of 228 ft long by $101\frac{2}{5}$ ft wide. Find the total distance around (perimeter) the Parthenon. Leave your answer as a fraction, in feet.

Perimeter = ____________________________

2. Karen's favorite recipe for barbecue sauce calls for $2\frac{1}{3}$ cups of tomato sauce. The recipe makes enough barbecue sauce to serve 7 people. How much tomato sauce is needed for 1 serving?

1 serving = _______________ cup(s) of tomato sauce

3. Lucas recently went apple-picking. Then, he found the following recipe for apple crisp. The recipe serves eight people. On Monday Lucas decided to make an apple crisp for his family of four. It turned out so delicious, the following night he decided to make an apple crisp for his entire class, for a total of 24 students. Take half of each ingredient in the recipe for Monday, and then triple each ingredient for Tuesday. Write your answers as fractions.

Apple Crisp	**Monday**	**Tuesday**
$4\frac{1}{2}$ cups peeled, cored and sliced apples		
2 teaspoons lemon juice		
2 tablespoons water		
$\frac{1}{2}$ cup honey		
1 teaspoon ground cinnamon		
$\frac{3}{4}$ cup brown sugar		
$\frac{3}{4}$ cup all-purpose flour		
$\frac{3}{4}$ cup rolled oats		
4 tablespoons butter		

See the Answer page for the recipe's directions, in case you actually want to make it.

Math Must 4: Rational Numbers (Part 2)

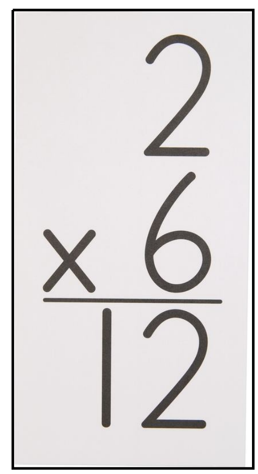
You've heard that no two snowflakes are exactly alike. Similarly, no two people have the exact same fingerprints. We're all quite unique. God made us that way. Well, natural numbers have a sort of "fingerprint" also. It's not called that, though. Instead we say that every natural number has its own, unique prime factors. Let's see if this can be explained simply.

Math Terms Reviewed

Every math student ought to know the following four terms. They are the answers to corresponding math operations. Look familiar?

Adding:	when you add two numbers you get a →	SUM
Subtracting	when you subtract two numbers you get a →	DIFFERENCE
Multiplying	when you multiply two numbers you get a →	PRODUCT
Dividing	when you divide two numbers you get a →	QUOTIENT

Now that you recall those, here's another question. The answer to a multiplication problem is called a product, but what are those two numbers you multiply together called? They are *factors*.



$2 \times 6 = 12$ In this case, 2 and 6 are *factors*, whereas 12 is a *product*.

Prime Numbers

A natural number is said to be ***prime*** if its only factors are 1 and itself. So that means that the number 2 is prime, since it only has factors of 1 and 2. Likewise, the number 3 is prime, since it only has factors of 1 and 3. But the number 4 is not prime. It has factors of 1, 2, and 4. Continuing on, 5 is prime, but 6 isn't. Here is a list of the first 11 prime numbers:

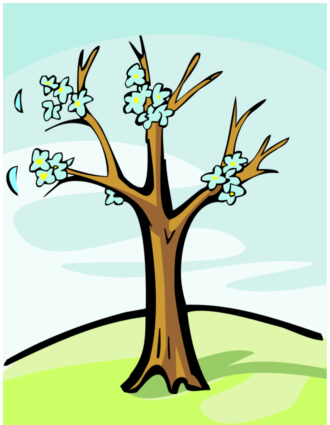
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ... the list goes on and on and on ...

Notice that the number 1 is not considered prime. The reason for this is beyond the scope of this lesson.

Composite Numbers

If a number isn't prime, it's called ***composite***. That means, if a number has factors other than 1 and itself, then it is a composite number.

The number 4 is composite because its factors are 1, 2, and 4. Likewise, 9 is composite since its factors are 1, 3, and 9.



Numbers Written in Prime Factored Form

Recall that factors are numbers that get multiplied together. So, writing a number in prime factored form means to "break down" a number into its prime factors, writing them out as a multiplication problem.

For example, the number 35 can be written as 5 × 7. Notice that both 5 and 7 are prime numbers. Here's another one. The number 12 can be written as 2 × 6. Although 2 is a prime number, 6 isn't. So, we must continue to "break down" the 6 further into its prime factors. And 6 can be written as 2 × 3. Now, since both of these are prime we can stop. Our final way to write 12 as prime factors is 12 = 2 × 2 × 3.

Try it again, using a factor tree:

12

/ \

<u>2</u> × 6

/ \

<u>2</u> × **<u>3</u>**

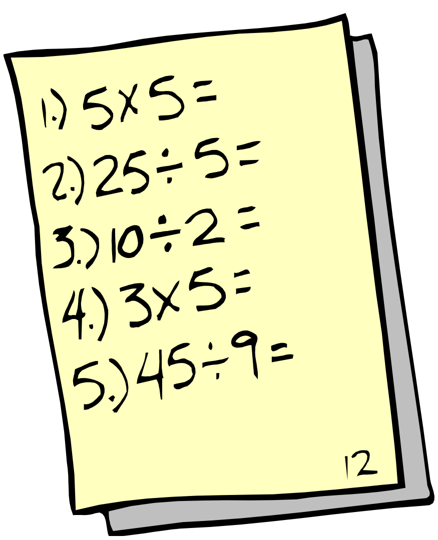
Notice that the prime numbers are **bold** and <u>underlined</u>. Or you could circle them. That helps to remind you that they cannot be "broken down" further.

So, the prime factors of 12 are 2 × 2 × 3.

Now, think about this for a minute. If you multiply the numbers 2 × 2 × 3, will you ever get a product other than 12? No way! So, 12 has a unique "fingerprint," if you will, of prime factors. Turns out, every natural number has a unique prime factor combination. Cool, huh?

Simplifying Fractions

You might be wondering, as fun as figuring out prime factors is, how helpful is this concept? Let me tell you. There are many uses for it, one of which is for simplifying fractions. That's another way to say "reducing" a fraction to its lowest terms.



So far all the fractions you've been shown have not required reducing. Reducing fractions requires several previously learned concepts—identity property, inverse property, multiplying fractions, and prime factors—all combined together. Here's an example of a fraction that *does* need to be reduced: $\frac{4}{6}$.

1) Start by writing the numerator and denominator as prime factors.

$$\frac{4}{6} = \frac{2 \times 2}{2 \times 3}$$

2) Now apply the rule for multiplying fractions in reverse. $\frac{2\times 2}{2\times 3} = \frac{2}{2}\times\frac{2}{3}$

3) Next, recall that a number divided by itself equals one (inverse property). $\frac{2}{2}\times\frac{2}{3} = 1\times\frac{2}{3}$

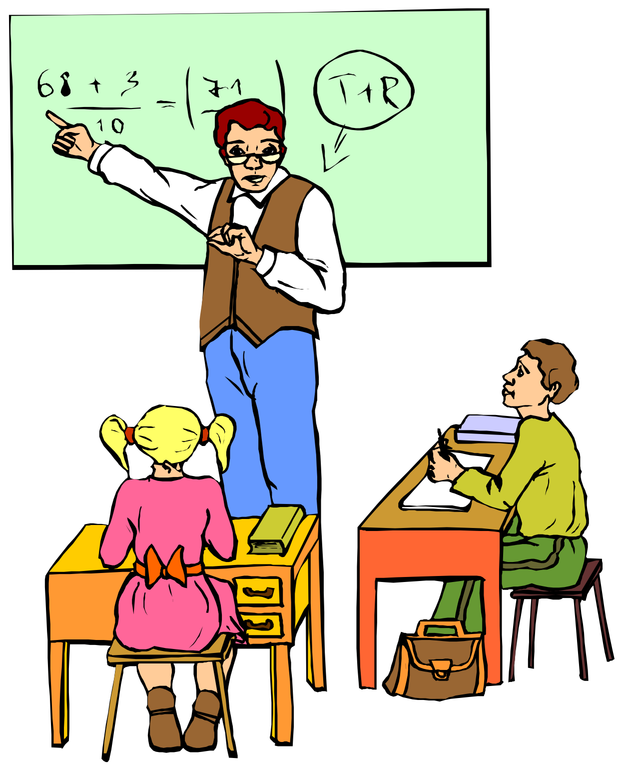
4) And finally, a number multiplied by one equals itself (identity property). $1\times\frac{2}{3} = \frac{2}{3}$

So $\frac{4}{6}$ reduces (simplifies) to $\frac{2}{3}$.

Here's another example: $\frac{24}{28}=\frac{2\times 2\times 2\times 3}{2\times 2\times 7}=\frac{2}{2}\times\frac{2}{2}\times\frac{2\times 3}{7}=1\times 1\times\frac{2\times 3}{7}=\frac{2\times 3}{7}=\frac{6}{7}$

One more example:

$$\frac{15}{75}=\frac{3\times 5}{3\times 5\times 5}=\frac{3}{3}\times\frac{5}{5}\times\frac{1}{5}=1\times 1\times\frac{1}{5}=\frac{1}{5}$$



Notice in this last example that the numerator has the number 1 in it. That is, everything else in the numerator canceled out, so since there isn't a number left, we place a 1 in the numerator to indicate that we still have a fraction, with a 5 in the denominator.

Name ______________________________ Date ______________

Practice: Rational Numbers (Part 2) Page 1

Use a prime factor tree to write the following numbers in prime factored form. Exercise 1 is started for you.

1. 90

/ \

9 × 10

90 = ______________________________

2. 63

63 = ______________________________

3. 48

48 = ______________________________

4. 300

300 = ______________________________

5. 425

425 = ______________________________

6. 885

885 = ______________________________

Name ______________________________ Date ______________

Practice: Rational Numbers (Part 2) Page 2

Simplify the following fractions. Begin by rewriting both the numerator and denominator as prime factors.

1. $\frac{12}{40}$

2. $\frac{105}{175}$

3. $\frac{44}{90}$

4. $\frac{280}{210}$

Perform the following operations; then simplify your answers.

5. $\frac{18}{7} \cdot \frac{9}{12}$

6. $\frac{21}{10} \div \frac{63}{24}$

7. $\frac{3}{18} + \frac{5}{18}$

8. $\frac{4}{5} - \frac{3}{12}$

Math Must 5: Order of Operations, Assumptions, & Basic Algebra

Ok, this is by far the weirdest title in this unit. But keep reading, you'll see why it is appropriately named.

It may seem like quite a leap, going from rather simple fractions to algebra. I think you're ready, though. Don't worry, we'll stick to the basics, the heart of algebra. At its core, algebra is all about finding the unknown. It's like a doing a puzzle without having a picture to go by, not knowing what the end result will be. There are guidelines (I call it a "recipe"), however, that will help you find what you're looking for. Follow them consistently, and you'll do just fine!

Before we get to those guidelines, though, let's get on the same page with regard to terminology and order of operations. First the terminology.

One of the most daunting things about higher math is all the unfamiliar words that are used—*coefficient, reciprocal, terms, abscissa, bases, constants, power, domain, and variables*—just to name a few. Not all of these words will be defined in this lesson, but we must start somewhere.

Base

Mathematicians, in general, don't like to write more than we have to. Shortcuts are our friends! Take the following multiplication, for example:

$$2 \times 2 \times 2 = 8$$

Obviously, the number two is multiplied by itself, three times, which equals 8. Another way to write the same thing, using a shortcut is like this:

$$2^3 = 8$$

When it is written this way, (brace yourself, here come some math terms) the 2 is called the "base" and the 3 is called the "exponent." Perhaps you already knew that, so we're easing into this, then. The shortcut is that the exponent tells the base how many times it (the base) should be multiplied by itself. Simple.

Now we're ready for order of operations; then we'll go back to more terms.

Order of Operations
Look at the following and write down the answer or say it out loud.

$$5+6\times 2$$

Got the answer? Good. Usually, when I present this to a class, I have them think the answer in their heads, or write it down on paper, but do **not** blurt the answer out loud. Once everyone has the answer, I ask by a show of hands, “How many of you say the answer is 22?” Almost always, some hands are raised. Then I ask, “How many of you say the answer is 17?” A different set of hands go up. Which group correct? the first group or the second group?

Turns out the second group is correct. The answer is 17. But why? What’s wrong with 22? Actually, nothing is wrong with the answer of 22. It’s a legitimate answer. However, herein lies the real problem. There can’t be two different answers to one problem like this. Either the answer is 22 or it’s 17. It can’t be both!

Either the problem is solved by adding 5 and 6 first, which is 11, then multiplying 11 and 2, which is 22. Or, the problem is solved by multiplying 6 and 2 first, which is 12, then adding 5 and 12, which is 17.

Method 1 (1st Group’s method)	**Method 2 (2nd Group’s method)**
$5+6\times 2$	$5+6\times 2$
11×2	$5+12$
Answer: 22	Answer: 17

So we must decide which method to use; then we all must agree to consistently stick to that method, doing problems like this the same way each time. Well, that decision was already made for us. The math community at large—everyone in Rhode Island, and New England, and the United States, and around the world, for that matter—has decided to use “Method 2.”

This method has the following operational order:

1) Do anything inside these grouping symbols first—parenthesis ( ), braces { }, or brackets [ ]. Begin with the innermost symbol and work your way out. The operations performed inside the symbol, must be done in the following order:
2) Reduce any exponents.
3) From left to right, perform any multiplication or division in the problem.
4) From left to right, perform any addition or subtraction in the problem.

There it is, the Order of Operations. Short and sweet. Often times this order is taught using the mnemonic PEMDAS to help remember the order.

P	Please	Parentheses
E	Excuse	Exponent
M	My	Multiply
D	Dear	Divide
A	Aunt	Add
S	Sally	Subtract

One more thing; if a problem has a fraction in it, perform the order of operations in the numerator (top) first. Then, do the same in the denominator (bottom). In other words, keep them separate. If, when you are done, the fraction can be simplified, then go ahead.

Ready for an example? Let's simplify the following:

$$3\left[4^2+4(4+2)\right]$$

First, we must simplify what's inside the innermost grouping symbol. That's the (4 + 2). Easy, 6. Ok, now we have:

$$3\left[4^2+4(6)\right]$$

TIME OUT!

Pause. Time out! Do you notice the 4(6) inside the [ ]. What does that mean? Well, if you don't see a + (plus) sign or a – (minus) sign in between two numbers, in math, you can safely assume they want you to multiply (× or · , both symbols mean *times*).

Here's another thing you need to quickly get used to in mathematics—assumptions. There are times when you must assume some things. I will try to point them out to you as we go along. This is one of them. Assume multiplication when two numbers are right next to each other without a + or – separating them.

Un-pause. Time in! We have to keep working inside the [ ]. Before we do the multiplication, we must take care of the exponent. So, four-squared is 16. Now we have:

$$3\left[16+4(6)\right]$$

Now we can multiply the 4 and 6 inside the [ ]. Twenty-four.

$$3[16+24]$$

We're getting somewhere. The problem is getting smaller. Next, we add 16 and 24.

$$3[40]$$

Do you remember what to do with two numbers without a plus or minus sign between them? Right, assume multiplication.

Answer: 120

Great job! There are more of these to try out in the practice problems found at the end of this lesson. Let's move on to more terms.

More Terms

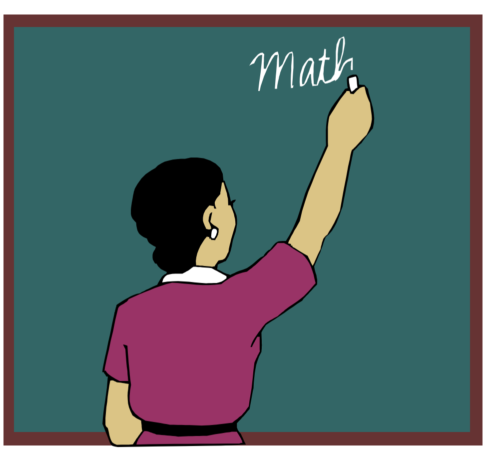
Earlier I showed you that $2^3 = 8$. We called 2 the base and 3 the exponent. We could also use *variables* as bases. We're getting suspiciously close to algebra. Well, that is where we're heading. When we want to represent a number, but are not quite sure what that number is, we use variables. Variables are letters (usually just a single letter, with *x* being the most popular) to represent, or stand for, the unknown number. Thus the root word "vary." The variable could be any number –8, 34, –5, ½ , who knows?

Here's a variable as a base. x^5

In this case, the base is *x*, and the exponent is 5. Which brings us to our next new term. This one's interesting. Take a look at this:

$$4x^3$$

You know what *x* is called, and what 3 is called, but what about the 4? Here's the new term—*coefficient*. A coefficient is a fancy math word for "the number out front." Yep, it's that easy. In this case 4 is the coefficient—the number out in front of the variable.



Let me explain two more assumptions you must understand. Remember this? x^5
What's the coefficient here? There appears to be no number out front. Well, when there appears to be no coefficient, there actually is one.

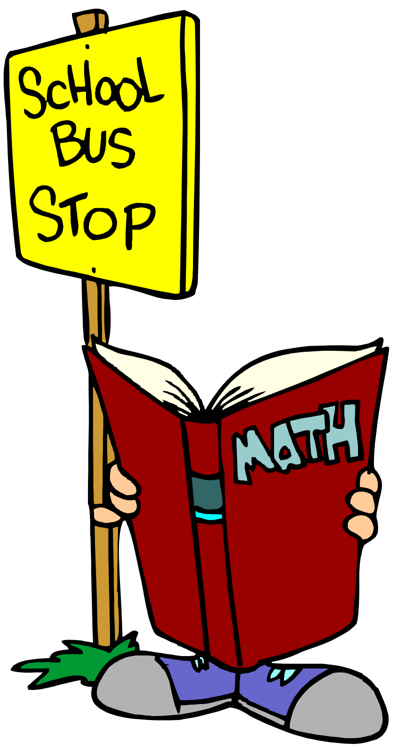
Oh the joys of math! Yes, there really is a coefficient. It's an understandable one (1).

Speaking of the number one. Look at this: $-7x$. You know that -7 is the coefficient, and x is the base. But what's the exponent? That's right, it is an understandable one (1). An unwritten exponent is going to be assumed to be one (1).

One more thing to explain before we get to algebra—*like terms*. Well, let's first discuss what a *term* is. A term is a combination of coefficient and / or variable. Typically, terms are separated by a + or a – sign. Here are some examples of groups of terms:

$$4x^6 + 7x \qquad\qquad -2x^7 - 9y^2 + 18 \qquad\qquad \frac{1}{3}a$$

The first group has two terms in it , the second group has three terms in it, and the last group has one term in it. I think the first example is pretty straight forward. But the middle set of terms is a bit more interesting. Notice the 18 at the end? It doesn't have a variable with it. That's ok. The 18 is still considered a coefficient, the reason for which is not covered in this unit. But, here's a good place to introduce another math term—*constant*. A constant is a better word to use for a coefficient without a variable, like 18 in our example.



To recap, the middle group of terms has three terms, one of which is a constant. A constant is still a term.

So, then, what are *like terms*? Like terms are those terms that have the same variables in them. More specifically, the variables that have the same exponents. For example, all of these are like terms:

$$5x^3 \qquad 2x^3 \qquad -13x^3 \qquad x^3 \qquad -0.25x^3 \qquad \frac{3}{7}x^3$$

Since each term has the same variable, and its exponent is the same for each, then they're like terms. Contrast those with the following terms, which are *unlike terms*:

$$5x^3 \qquad\qquad 2x^4 \qquad\qquad -13x \qquad\qquad 44$$

Are you still with me? Have I lost you along the way? If you're confused, you might want to stop reading for a while. Come back to this in about 10-15 minutes. Not much longer, because then you'll forget why you stopped in the first place.

Back now? Good. Now back up and reread the last several paragraphs to just make sure you've got all the correct terminology. When we get to the algebra stuff, you're going to need to know what these words mean.

Basic Algebra

You've arrived. Here it is—the big time—algebra! Remember my description of the heart of algebra? It's like doing a puzzle where you are trying to put the pieces together, but without knowing what it's supposed to look like.

The gist of algebra is solving an equation for the missing variable. Or simply put, if you are given an equation, find out what number the variable represents so that the left-hand side of the equal sign is the same as the right-hand side. Let me show you a simple example.

Solve: $x+5=9$

This means, some number, *x*, if added to 5 equals 9. What is that number? What is *x*? Obviously the number is 4. So, *x* = 4. You don't really need to know too many rules to solve this one. It can be solved just by looking at it and using your God-given brain. But what about this next equation?

Solve: $7(p-2)+p=2(p+2)$

Uh oh. Not so easily solved, is it? We need a set of rules to help out. Or better yet, let me give you what I call a "recipe" for solving equations like these. Just like if you try to cook or bake something for the first time, you probably use a recipe to assure that the dish comes out the way it should. Well, here's a "recipe" that if you follow step by step, you will find the answer you're looking for—you'll get the "dish" or "baked goods" of algebra.

By the way, don't let the title scare you. I'm throwing in the word *linear*, because if you were to graph these equations (which is also beyond the scope of this unit), then it would look like a straight line.

"Recipe" for Solving Linear Equations

1. Get rid of ( ).
2. Add up like terms on each side of the equal sign.
3. Get "*x*" onto one side of the equal sign.
4. Get rid of the constant term on the same side as "*x.*"
5. Get rid of the coefficient of "*x,*" if not already 1.

Keep in mind that it's possible, while solving these equations, that you might not have to do anything for one or more of these steps. For example, when you solved the simple equation, x + 5 = 9, you actually used the recipe without even knowing it. But not every step of the recipe was used. In fact, only one of them was used—Step 4.

Let's walk through each of the steps of the recipe and understand what's involved in each one. We'll solve our new equation at the same time.

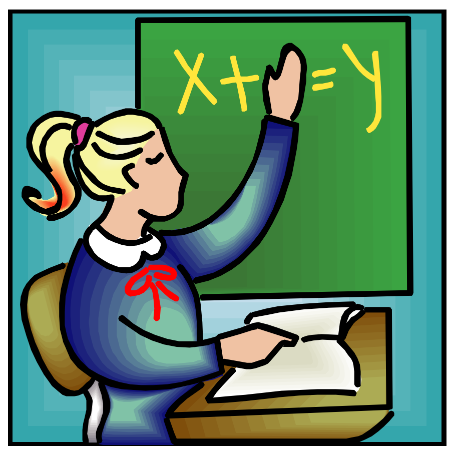
$$7(p-2)+p=2(p+2)$$

Step 1: Get rid of ( ). If the equation starts off with parentheses in them, to get rid of them, we use the distributive property we learned in Math Must 2: Properties of Real Numbers. That is, each term inside the ( ) gets multiplied by the number in front of the ( ). Once you use the distributive property, the ( ) are gone.

$$7p-14+p=2p+4$$

Step 2: Add up like terms on each side of =. In our example the right-hand side does not have any like terms to add together. But the left-hand side does. The 7*p* and the *p* are like terms. To add them, keep the variable the same, simply add the coefficients together. So, 7*p* + *p* equals 8*p*. It's like saying 7 pencils plus 1 pencil equals a total of 8 pencils. Get it? "*p*" for pencil.

$$8p-14=2p+4$$



Step 3: Get "*x*" onto one side of =. Notice I wrote *x* in quotes. That's because, *x* isn't always the variable of interest in the equation. In our example we're looking for *p*. So, we need to get *p* onto one side of the equal sign. Right now, it's on both sides. It's on the left side with 8, and on the right side with 2.

We need to pause here for a bit. There's something you must know about equations before proceeding. You may or may not already know this, but here goes.

An equation is like a teeter-totter that is perfectly balanced on both sides. The same amount of weight is on both sides of the teeter-totter so that it's "even" on both sides. Both sides are at the same height off the ground, so to speak. So, if you want to add or take away weight from one side of the teeter-totter, to keep it balanced you must add or subtract the same amount of weight to the other side, right? Likewise, if you want to double (multiply times 2) the weight of one side, you must do the same thing to the other side, to keep things equal. The same goes for equations. Each side of an equation wants to be treated the same! Simply put, whatever you do to one side of the equation, you must do to the other.

Ok, un-pause. Back to our example. We wanted to get *p* onto one side of the equation. How about if we subtract 2*p* from the right-hand side? That would make it disappear from the right-hand side. Why? Which property, learned in a previous lesson, is at work here? But wait, we subtract 2p from one side, we must do the same to the other side, right? to keep things balanced?

$$8p-14-2p=2p+4-2p$$

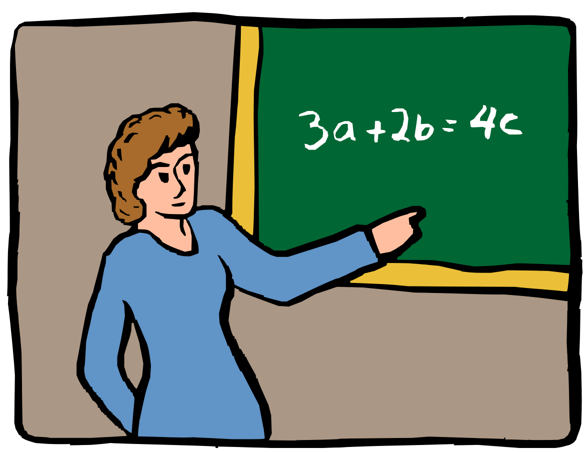
$$6p-14=4$$

Step 4: Get rid of constant term on same side as "*x.*" First off, do you see two different constant terms? There's the –14 on the left-hand side and the 4 on the right-hand side. Since there might be more than one constant, I added the part about the one on the same side as "*x*" to help out. In our example, the one to get rid of is –14 since it is on the same side as our variable. To get rid of it, let's add 14 to both sides of the equation.

$$6p-14+14=4+14$$

$$6p=18$$

Step 5: Get rid of coefficient of "*x.*" Remember that since we do not see a + sign or – sign in between 6 and *p*, we assume multiplication. And the opposite operation of multiplication is division. So, let's divide both sides by 6. Doing so finally get *p* all by itself with a coefficient of an understandable 1. Just this one step alone uses multiple topics learned in previous lessons! Dividing both sides by 6 makes the left-hand side uses both the inverse property and the identity property.



$$\frac{6p}{6}=\frac{18}{6}$$

$$p=3$$

Great! We found the solution! So, if we replace every *p* in the original equation with 3, then simplify each side, we'll get the left-hand side to look exactly like the right-hand side. This is one of the benefits of math you just have to take advantage of—checking your answer to see if you got it correct.

$$7(p-2)+p=2(p+2)$$
$$7(3-2)+3=2(3+2)$$
$$7(1)+3=2(5)$$
$$7+3=10$$
$$10=10$$ ✓ Check! It works!

Name ______________________________ Date ____________

Practice: Order of Operations, Assumptions, & Basic Algebra Page 1

Using the correct order of operations, find the value of the following expressions.

1. $13+9\cdot 5$ = __________

2. $11\cdot 4+10\cdot 3=$ __________

3. $12+8^2\div 8-4=$ __________

4. $30-3(4+2)=$ __________

5. $5[3+4(2^2)]=$ __________

6. $\dfrac{8+6(3^2-1)}{3\cdot 2-2}=$ __________

7. $\dfrac{4(7+2)+8(8-3)}{6(4-2)-2^2}=$ __________

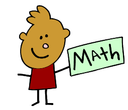
8. $\dfrac{4^2[(13+4)-8]}{18-2(3+4)}=$ __________

Add the like terms together in each expression.

9. $16y+5b+10y-3b$ = ________________

10. $12x^2+7x-2x-8x^2+9x=$ ________________

11. $43a+22t-14m^2-9t+10a+27m^2+6x+8a$ = ________________________



Name ____ Date ________________

Practice: Order of Operations, Assumptions, & Basic Algebra

Solve the following equations. If necessary, leave non-integer answers as fractions.

1. $x - 4 = 8$ x = _______

2. $r + 9 = 13$ r = _______

3. $3x = 2x + 7$ x = _______

4. $8x - 4 = 6 + 7x$ x = ________

5. $4x - 3 - 8x + 1 = -5x + 9$

   x = _______

6. $2(p + 5) - (9 + p) = -3$

   p = _______

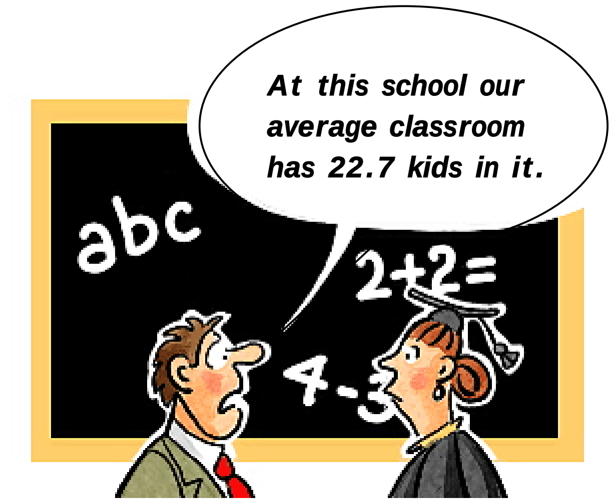
7. $2 - 3(2 + 6m) = 4(m + 1) + 18$

   m = _______

8. $9(2t - 3) - 4(5 + 3t) = 5(4 + t) - 3$

   t = _______

Math Must 6: Decimals and Percents



Hopefully now you feel more comfortable with fractions! Let's discuss decimals and percents next. Recall that any number that can be written as a fraction is called a rational number. Decimals and percents are considered rational numbers, because they're in the fraction family of numbers.

In this lesson we're going to convert decimals or percents into fractions, since we now know how to handle fractions. Sure, we could use a calculator to deal with decimals and percents, but since you're now a fraction expert (this is the part where you chuckle to yourself) let's treat them as fractions. Besides, what if you are stranded all by yourself on a deserted island and you forgot to bring your calculator with you? I know what you're thinking…"Why would I be doing math problems involving decimals or percents if I were alone on an island?" Perhaps you are trying to figure out the probability of getting rescued off of the island. Regardless, here goes!

<u>Decimals</u>
The simplest of decimals are those which end. Take 0.52, for example. This is a decimal number that has 2 digits after the decimal point (decimal). The decimal digits do not continue on and on forever—they stop. So, to convert a decimal number into its fractional equivalent, you simply move the decimal to the right, counting the number of digits you passed, until you get to the end.

In our example you would move the decimal 2 places to the right to convert 0.52 into 52. Next, place your new number over a 1 followed by zeros. The number of zeros you should write will be the same as the number of digits you passed when you moved the decimal. Our new fraction becomes:
$\frac{52}{100}$. So, 0.52 = $\frac{52}{100}$. Not too bad, huh?

One more thing, if the fraction can be simplified we must do that to complete the fraction. So,
$0.52 = \frac{52}{100} = \frac{2\cdot 2\cdot 13}{2\cdot 2\cdot 5\cdot 5} = \frac{13}{5\cdot 5} = \frac{13}{25}$.

In Math Must 1: Sets of Numbers, we began to explain the next kind of decimal numbers to look at—decimals that don't stop, but continue on and on forever, with a noticeable pattern. Take 0.5555…, for example. This decimal contains a countless number of 5's after the decimal. It never stops. You would not be able to use the same process explained above for a decimal like this. How many zeros would you write in the denominator?

The trick lies in the number 9. Here's the mathematical explanation: let X represent the decimal number that repeats forever.

$$X = 0.55555\ldots$$

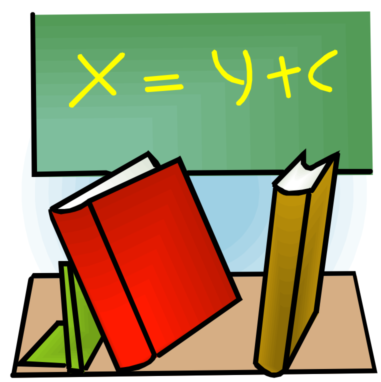
If you multiplied X by 10 in our equation, the left-hand side would be 10X, and the right-hand side would be 5.5555…

$$10X = 5.5555\ldots$$

Notice that the pattern has only one digit that keeps repeating over and over. So that's why I multiplied the left-hand side by 10. The number of zeros you use corresponds with how many digits are repeated. Let's continue; then we'll do another example.

Next, subtract X from 10X, as shown:

$$\begin{array}{rr} & 10X = 5.5555\ldots \\ \underline{-} & \underline{X = 0.5555\ldots} \\ & 9X = 5.0000\ldots \end{array}$$



The left-hand side is 10X – X, which equals 9X. This is actually algebra! And the right-hand side equals 5. Notice that all the 5's after the decimal cancel each other out, or become zero!
Thus, we have:

$$9X = 5$$

In algebra, to find X in this case, we would divide both sides of the equal sign by 9, giving us: $X = \frac{5}{9}$. With a calculator, divide 5 by 9 and see what decimal answer you get. So,

$$0.55555\ldots = \frac{5}{9}$$

Let's try one more example. Convert the decimal 0.123123123… into its fractional equivalent. Start by setting the decimal equal to X, then multiply X by 1000. This time the repeating pattern has 3 digits in it. That's why we multiply X times 1000 (three zeros).

$$\begin{array}{rl} 1000X = & 123.123123123\ldots \\ \underline{-\quad X =} & \underline{\quad 0.123123123\ldots} \\ 999X = & 123 \end{array}$$

$X = \frac{123}{999}$ simplified ➔ $X = \frac{41}{333}$ so ➔ $0.123123\ldots = \frac{41}{333}$

Shortcut

Do you remember, from Math Must 1: Sets of Numbers, that the trick lies with the number 9? To get a repeating decimal, place as many 9's in the denominator as there are digits that repeat in the decimal.

So, if the decimal is 0.67676767…, then the denominator will be 99, since two digits repeat. And if the decimal is 0.123451234512345…, then the denominator will be 99999, since five digits repeat. That's the shortcut!

Percents

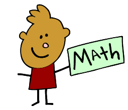
A good place to start with percents is what the word itself means. "Per" means *by* or *over*. And "centum" is the Latin word for 100, as in the word *century* (100 years). Putting the two meanings together, you get "over 100" or "by the 100." And that's exactly what a percent is. It's a number that can be written over 100, as a fraction.

Let's say you took a quiz or a test, and your percent score is 89%. As a fraction your score is $^{89}/_{100}$. As a decimal, that's 0.89. There it is. That's what percents are all about. Too easy, you say? True. Let's make it a bit more complicated.

Before I keep going, let me define two very important, highly technical, mathematical terms: "of" and "is." You may be thinking, what makes these words so important, so technical? Well, turns out we actually use these words quite often in math. Perhaps you used them today without even realizing it. Here are some examples of when these words are used:

"What is 4 times 9?" *(Do you see one of the words?)*

"Oh no! Half of the class failed the test!" *(Do you see the other word?)*



30%off

"Hey, this shirt is on sale. What is thirty percent of forty-eight dollars?"
(Do you see both of the words this time?)

What do the words mean? In math, specifically with word problems, the word *is* means "equals," and the word *of* means "times" or "multiplied by." Here are the phrases again, but with their equivalent equations next to them.

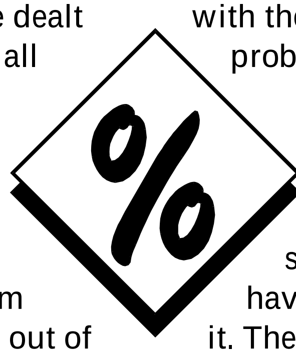
"What *is* 4 times 9?" ➔ what # = 4 × 9

"Half *of* the class failed the test!" ➔ ½ × total # of students in class = # that failed

"What *is* thirty percent *of* forty-eight dollars?" ➔ what # = 30% × $48

Now that you have a better idea as to what the terms mean, let's look at how to deal with percents. All percent problems can be dealt with the same way. That is, use the following general equation to handle all problems with percents in them.

$$\frac{P}{100}\times(\text{of}) = \text{is}$$



Here's how we're going to use this simple, general equation. Any time you see or read the part of the problem having to do with "percent," put that number over 100 and make a fraction out of it. The part of the problem after the word "of," get multiplied by the percent fraction. And the number we are looking for, or that comes after the word "is," gets placed on the right-hand side of the = sign.

We're going to call the *P* the "percent" part of the problem. That's highly appropriate, don't you think? Let's call the "of" part of the problem the "whole." And finally let's call the "is" part of the problem our "answer." So the product of the percent and the whole gives us the answer.

Let's try a problem. We'll solve for the percent.

What percent of 8300 is 996?

Here is how to convert the question into a percent problem:

$$\frac{P}{100}\times 8300 = 996$$

I hope this makes sense to you. Can you see how the percent part was put over 100? The word "of" was changed to multiplication? Followed by the 8300 that comes after "of"? Followed by an equal sign (=) to replace the word "is"? And then we write the 996 after the word "is"?

Now what? Well, we use the "recipe" in algebra to help us solve for *P*. Let's get rid of the 8300 from the left-hand side. Since the percent is multiplied by 8300, dividing both sides by 8300 is the opposite operation.

$$\frac{P}{100} \times 8300 \div 8300 = 996 \div 8300$$

$$\frac{P}{100} = \frac{3}{25}$$

We've almost got *P* all by itself. The final step is do multiply both sides by 100, doing the opposite to what we see on the left-hand side. Multiplying both sides by 100 gets rid of the 100 on the left-hand side.

$$\frac{P}{100} \times 100 = \frac{3}{25} \times 100$$

$$P = \frac{300}{25}$$

$$P = 12$$

So, 12% of 8300 is 996. That's the answer to our problem.

Here's another simple problem. This time we'll solve for the "answer." The set-up is slightly different. But we can still solve it if we obey the rules and term translations we've already learned. This problem is so simple you can, I'm sure, solve it just by looking at it. Is it obvious to you that 50% of 800 is 400? Sure, one-half of 800 is 400.

What is 50 percent of 800?

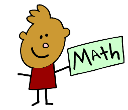
$$WN = \frac{50}{100} \times 800$$

$$WN = \frac{40000}{100}$$

$$WN = 400$$

Here, *WN* stood for "what number." So, the number we were looking for, and what we expected it to be, was 400.

In the first examples we solved for the "percent." Then in the second example we solved for the "answer." In our last example, we're going to solve for the "whole."



Eighty percent of what number is 1120?

Here we want to know the "whole" part of the equation. The part following the word "of."

$$\frac{80}{100} \times WN = 1120$$

Multiplying both sides by 100 and dividing both by 80, gives us:

$$\frac{80}{100} \times WN \times \frac{100}{80} = 1120 \times \frac{100}{80}$$

$$WN = \frac{112000}{80}$$

$$WN = 1400$$

Final answer 1400.

Converting from Decimal to Percent or Percent to Decimal

One final thing about percents before we're done. Sometimes we must convert from a percent to a decimal, or vice versa. The decimal point is all we need to worry about. We have to move it two places to the right, or two places to the left. Which is it? and when?

Here's an easy way to remember what to do. First, write down these two words: decimal, percent. Notice I wrote decimal first, then percent second. I wrote them in that order, because they're in alphabetical order ("d" comes before "p" in the alphabet).

Decimal	Percent
to change to percent move decimal two places to the right →	to change to a decimal move decimal two ← places to the left

Name ____ Date ________________

Practice: Decimals and Percents Page 1

Convert the following decimals into simplified fractions.

1. 0.8 = __________

2. 0.431 = ___________

3. 20.58 = ___________

4. 3.805 = ___________

Convert the following repeating decimals into simplified fractions.

5. $0.66666...$ = ___________

6. $.252525...$ = ___________

7. $1.56675667...$ = ___________

8. $0.83333...$ = ___________

9. Create your own repeating decimal pattern, so that three digits repeat:

____

Convert the decimal you created into a simplified fraction: _______________

10. Create your own repeating decimal pattern, so that four digits repeat:

____

Convert the decimal you created into a simplified fraction: _______________

11. Create your own repeating decimal pattern, similar to problem #7 or #8, such that it has a leading number that does not repeat, followed by a repeating decimal:

____

Convert the decimal you created into a simplified fraction: _______________

Name ____ Date ________________

Practice: Decimals and Percents Page 2

Convert the following decimals into percents.

1. 0.45 = __________%
2. 0.06 = __________%
3. 0.003 = __________%
4. 1.12 = __________%

Convert the following percents into decimals.

5. 22% = __________
6. 7.3% = __________
7. 0.04% = __________
8. 100% = __________

Solve the following for the missing value.

9. What percent of 68 is 95.2? ______________________

10. What percent of 98 is 3.92? ______________________

11. Fifteen percent of what number is 10.5? ______________________

12. Sixty-five percent of what number is 260? ______________________

13. Three hundred seventy-five percent of 1300 is what number? ______________________

14. What number is thirty-three percent of 700? ______________________

Answers to Practice Problems:

page 9

1. Naturals {4}; Wholes {0, 4};
   Integers {–5, –1, 0, 4};
   Rationals {–5.3, –5, –1, –1 / 9, 0, 1.2, 4};
   Irrationals $\{-\sqrt{3}, \sqrt{12}\}$;
   Reals {–5.3, –5, –1, –1 / 9, 0, 1.2, 4, $-\sqrt{3}, \sqrt{12}$}
2. *Note: answers already given are not shown.*
   (10) yes, yes; (-2) no, yes, yes;
   (1 / 3) no, yes, no, yes;
   ($-\sqrt{3}$) no, no, no, yes, yes;
   (2.5) no, no, no, yes; (π / 2) no, yes, no, yes;
   ($\sqrt{100}$) yes, yes, no, yes

page 10

1. $\pi \approx$ 3.14159265358979323846 26…
2. (a) 3.1415926; (b) 3.141592653589;
   (c) 3.14159265358979
3. answers will vary

page 11

Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
Note: the following answers are approximates.
1. 2.2
2. 3.2
3. 3.9
4. 7.1
5. 6.3
6. 8.4
7. 9.1
8. 5.7
9. 9.6
10. 10.9

page 15

1. Identity; 2. Associative; 3. Inverse;
4. Commutative; 5. Distributive; 6. Inverse;
7. Commutative; 8. Inverse; 9. Distributive;
10. Identity

page 16

1. no; 2. no; 3. yes; 4. answers will vary;
5. answers will vary; 6. create a messy room;
7. spending money; 8. turning the volume down on your iPod; 9. answers will vary

page 17

1. Identity Property
2. a woman who is afraid of her husband; a husband who fears women (or respects them)
3. a man actually biting a dog; a dog that bites people
4. answers will vary

page 21

1. 24 / 35	2. 50 / 63	3. 49 / 3
4. 21 / 20	5. 10 / 3	6. 3 / 20
7. 7 / 12	8. 3 / 5	9. 8 / 9
10. 7 / 15	11. 17 / 12	12. 29 / 30

page 22

1. $658\frac{4}{5}$ ft *or* $\frac{3294}{5}$ ft   2. 1 / 3

3. Monday: $9/4$ or $2\frac{1}{4}$ cups; 1 tsp; 1 tbsp; $1/4$ cup; $1/2$ tsp; $3/8$ cup; $3/8$ cup; $3/8$ cup; 2 tbsp; Tuesday: $27/2$ or $13\frac{1}{2}$ cups; 6 tsp; 6 tbsp; $3/2$ or $1\frac{1}{2}$ cups; 3 tsp; $9/4$ or $2\frac{1}{4}$ cups; $9/4$ or $2\frac{1}{4}$ cups; $9/4$ or $2\frac{1}{4}$ cups; 12 tbsp

Apple Crisp Baking Directions:
1) Preheat oven to 350°F. Lightly grease a medium casserole dish (approximately 9" square).
2) Evenly spread the apple slices in the prepared dish. In a small bowl, mix the lemon juice and water, and pour over the apples. Drizzle apples with honey, and sprinkle with cinnamon.
3) In a bowl, mix the brown sugar, flour, oats, and butter until the mixture resembles coarse crumbs. Sprinkle over the apples.
4) Bake 25 minutes in the preheated oven, until apples are tender and topping is lightly browned.

Answers to Practice Problems: *(cont.)*

page 26

1. $2 \cdot 3 \cdot 3 \cdot 5$
2. $3 \cdot 3 \cdot 7$
3. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
4. $2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$
5. $5 \cdot 5 \cdot 17$
6. $3 \cdot 5 \cdot 59$

page 27

1. 3/10	2. 3/5
3. 22/45	4. 4/3
5. 27/14	6. 4/5
7. 4/9	8. 11/20

page 36

1. 58	2. 74
3. 16	4. 12
5. 95	6. 14
7. 19/2	8. 36

9. $26y + 2b$
10. $4x^2 + 14x$
11. $61a + 13t + 13m^2 + 6x$

page 37

1. 12	2. 4
3. 7	4. 10
5. 11	6. –4
7. $-\frac{13}{11}$	8. 64

page 44

1. $\frac{4}{5}$	2. $\frac{431}{1000}$
3. $\frac{1029}{50}$	4. $\frac{761}{200}$
5. $\frac{2}{3}$	6. $\frac{25}{99}$
7. $\frac{5222}{3333}$	8. $\frac{5}{6}$

9. answers will vary
10. answers will vary
11. answers will vary

page 45

1. 45%	2. 6%
3. 0.3%	4. 112%
5. 0.22	6. 0.073
7. 0.0004	8. 1 *or* 1.0

9. 140%
10. 4%
11. 70
12. 400
13. 4875
14. 231

Unit Resources:

Algebra 1: An Incremental Development; 2nd edition; John H. Saxon, Jr.; Saxon Publishers; © 1990.

Introductory Algebra; 8th edition; Lial, Hornsby, & McGinnis; Pearson Addison Wesley Publishing; © 2006.

Mathematical Ideas; 7th edition; Miller, Heeren, & Hornsby; HarperCollins College Publishers; © 1994.

http://mathforum.org/dr.math/

Online Resources:

- http://www.algebrahelp.com/
  - Algebrahelp.com is a collection of lessons, calculators, and worksheets created to assist students and teachers of algebra.
- http://www.middleweb.com/CurrMath.html
  - This website has multiple links within it, pointing you to different websites that offer help with math, from lesson plans to math explanations. Fantastic resource!
- http://www.coolmath.com/
  - Games, puzzles, math dictionary. It's all here and really fun!
- http://www.qcc.mass.edu/mathsupport/arith.pdf
  - Fractions, decimals, & percents explained, with examples & answers to several problems.
- http://mathforum.org/dr.math/
  - Have a math-related question? Ask Dr. Math. At this website there's bound to be an answer to your question!
- http://www.sparknotes.com/math/
  - Free study guides for various math topics.
- http://www.manatee.k12.fl.us/sites/elementary/palmasola/mathlabtutorials.htm
  - Fun explanations for mathematical thinking.
- http://www.multcolib.org/homework/mathhc.html
  - Another website chuck-full of math help and resources.
  - Want to know how an abacus works? Need geometry flash cards? It's here!

Online Resources: *(cont.)*

- http://www.pbs.org/teachers/math/
  - Lesson plans for multiple topics and grades
- http://www.edu4kids.com/
  - Click to the left-hand side on the link for "Math General" for flash cards and timed games.
- http://www.mathsisfun.com/
  - More fun with math explanations and games.