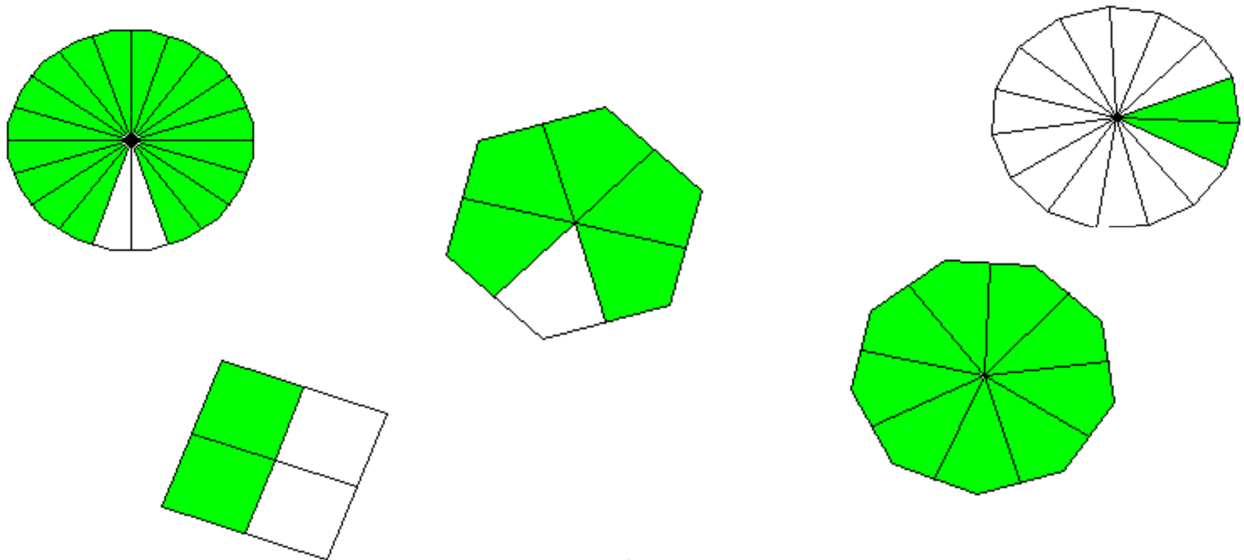
Fantastic Fractions



An

Integrated Unit of Study

by

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Major Concepts

Basic Fractions
Fourths, Eights, and Tenths
Adding Fractions
Comparing Fractions With Same Denominators
Comparing Fractions With Unlike Denominators
Comparing Unlike Fractions
Comparing Decimals and Fractions
Converting Fractions to Mixed Numbers
Converting Fractions to Percents
Converting from Percents
Converting to Decimals
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Dividing Mixed Numbers
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Subtracting Fractions – Different Denominators
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The Definition

Dictionary

Definition frac·tion

[Lat.,=breaking], in arithmetic, an expression representing a part, or several equal parts, of a unit.

Antonyms: entirety, total, whole

Mathematics.

- An expression that indicates the quotient of two quantities, such as 1/3.
- A disconnected piece; a fragment.
- A small part; a bit: *moved a fraction of a step.*
- A chemical component separated by fractionation.

World History

Our word fraction did not originally have a mathematical sense. It goes back ultimately to the Latin verb frangere, "to break." From the stem of the past participle frāctus is derived Late Latin frāctiō (stem frāctiōn-), "a breaking" or "a breaking in pieces," as in the breaking of the Eucharistic Host. In Medieval Latin the word frāctiō developed its mathematical sense, which was taken into Middle English along with the word. The earliest recorded sense of our word is "an aliquot part of a unit, a fraction or subdivision," found in a work by Chaucer written about 1400. One of the next recorded instances of the word recalls its origins, referring to the "brekying or fraccioun" of a bone.

Notation for Fractions

In writing a fraction, e.g., 2⁄5 or $\frac{2}{5}$, the number after or below the bar represents the total number of parts into which the unit has been divided. This number is called the denominator. The number before or above the bar, the numerator, denotes how many of the equal parts of the unit have been taken. The expression 2⁄5, then, represents the fact that two of the five parts of the unit or quantity have been taken. The present notation for fractions is of Hindu origin, but some types of fractions were used by the Egyptians before 1600 B.C. Another way of representing fractions is by decimal notation.

Characteristics of Fractions

When the numerator is less than the denominator, the fraction is <u>proper</u>, i.e., less than unity. When the reverse is true, e.g., $\frac{5}{2}$, the fraction is <u>improper</u>, i.e., greater than unity. When a fraction is written with a whole number, e.g., $3\frac{1}{2}$, the expression is called a <u>mixed number</u>. This may also be written as an improper fraction, as $\frac{7}{2}$, since three is equal to six halves, and by adding the one half, the total becomes seven halves, or $\frac{7}{2}$. A fraction has been reduced to its lowest terms when the numerator and denominator are not divisible by any common divisor except 1, e.g., when $^4/_6$ is reduced to $\frac{2}{3}$.

Arithmetic Operations Involving Fractions

When fractions having the same denominator, as $\frac{3}{10}$ and $\frac{4}{10}$, are added, only the numerators are added, and their sum is then written over the common denominator: $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$. Fractions having unlike denominators, e.g., $\frac{1}{4}$ and $\frac{1}{6}$, must first be converted into fractions having a common denominator, a denominator into which each denominator may be divided, before addition may be performed. In the case of $\frac{1}{4}$ and $\frac{1}{6}$, for example, the lowest number into which both 4 and 6 are divisible is 12. When both fractions are converted into fractions having this number as a denominator, then $^1/_4$ becomes $\frac{3}{12}$, and $\frac{1}{6}$ becomes $\frac{2}{12}$. The change is accomplished in the same way in both cases—the denominator is divided into the 12 and the numerator is multiplied by the result of this division. The addition then is performed as in the case of fractions having the same denominator: $\frac{1}{4} + \frac{1}{6} = \frac{3}{12} + \frac{2}{12} = \frac{5}{12}$. In subtraction, the numerator and the denominator are subjected to the same preliminary procedure, but then the numerators of the converted fractions are subtracted: $\frac{1}{4} + \frac{1}{6} = \frac{3}{12} + \frac{2}{12} = \frac{1}{12}$.

In multiplication the numerators of the fractions are multiplied together as are the denominators without needing change: $\frac{2}{3} \times \frac{3}{5} = {^6/_{15}}$. It should be noted that the result, here $^6/_{15}$, may be reduced to $^2/_5$ by dividing both numerator and denominator by 3. The division of one fraction by another, e.g., $^3/_5 \div {^1/_2}$, is performed by inverting the divisor and multiplying: $^3/_5 \div {^1/_2} = {^3/_5} \times {^2/_1} = {^6/_5}$. The same rules apply to the addition, subtraction, multiplication, and division of fractions in which the numerators and denominators are algebraic expressions.

A word from Wikipedia . . .

A fraction (from the Latin <u>fractus</u>, broken) is a number that can represent part of a whole.

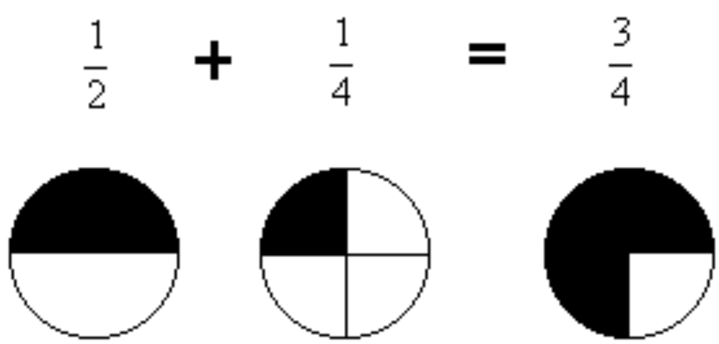
The earliest fractions were reciprocals of integers, symbols representing one half, one third, one quarter, and so on. A much later development were the common or "vulgar" fractions which are still used today, and which consist of a numerator and a denominator, the numerator representing a number of equal parts and the denominator telling how many of those parts make up a whole. An example is 3/4, in which the numerator, 3, tells us that the fraction represents 3 equal parts, and the denominator, 4, tells us that 4 parts make up a whole.

A still later development was the decimal fraction, now usually called simply a "decimal", in which the denominator is a power of ten, determined by the number of digits to the right of a decimal separator. In English-speaking and many Asian and Arabic-speaking countries, a period (.) or raised period (•) is used as the decimal separator. In most other countries, however, a comma is used. Thus in 0.75 the numerator is 75 and the denominator is 10 to the second power (because there are two digits to the right of the decimal). Thus the denominator is 100.

A third kind of fraction still in common use is the "per cent", in which the denominator is always 100. Thus 75% means 75/100.

Other uses for fractions are to represent ratios, and to represent division. Thus the fraction 3/4 is also used to represent the ratio 3:4 (three to four) and the division 3 ÷ 4 (three divided by four).

In mathematics, the set of all fractions is called the set of rational numbers, and is represented by the symbol Q.



Meet the Fractions

Fractions are expressed as one number over another number, like this:

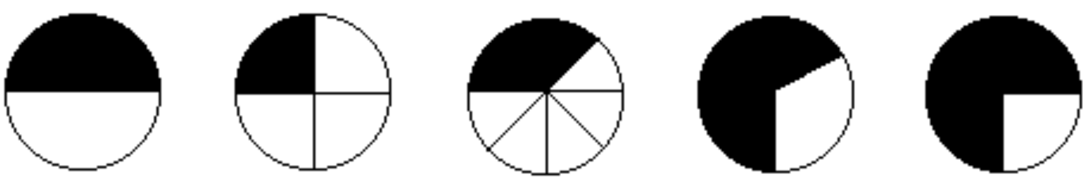
$$\frac{1}{2}$$

The number on the top is called the *numerator* and the number on the bottom is called the *denominator*.

EXAMPLE:
When you think of a fraction, think of a PIZZA!!

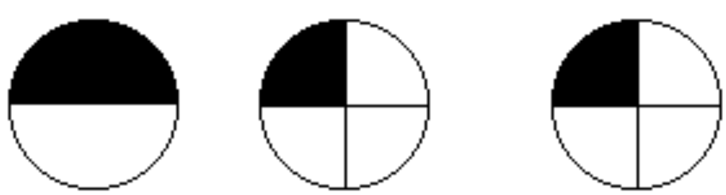
Suppose a pizza is cut evenly into the number of pieces in the DENOMINATOR. If the number of pieces YOU get is the NUMERATOR, the fraction of the pizza you get is:

$$\frac{1}{2} \qquad \frac{1}{4} \qquad \frac{3}{8} \qquad \frac{2}{3} \qquad \frac{3}{4}$$



Adding and taking away (subtracting) fractions can be pictured using slices of pizza. For example:

$$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

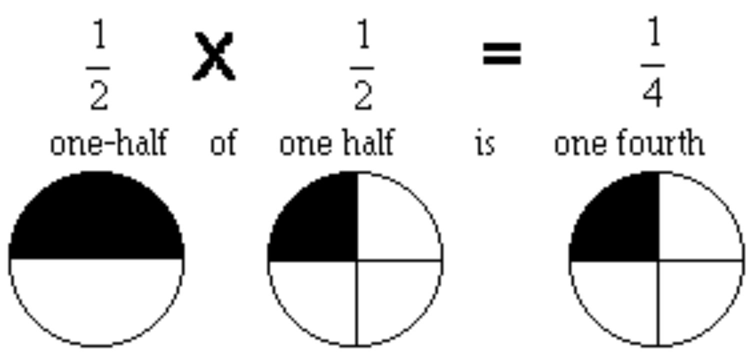


Multiplying fractions means cutting a portion into smaller portions. For example:

Dividing fractions means determining how many smaller pieces there are in a larger piece. For example:

$$\frac{1}{2} \div \frac{1}{4} = 2$$

This means there are 2 one-fourth pieces of pizza in a half pizza.

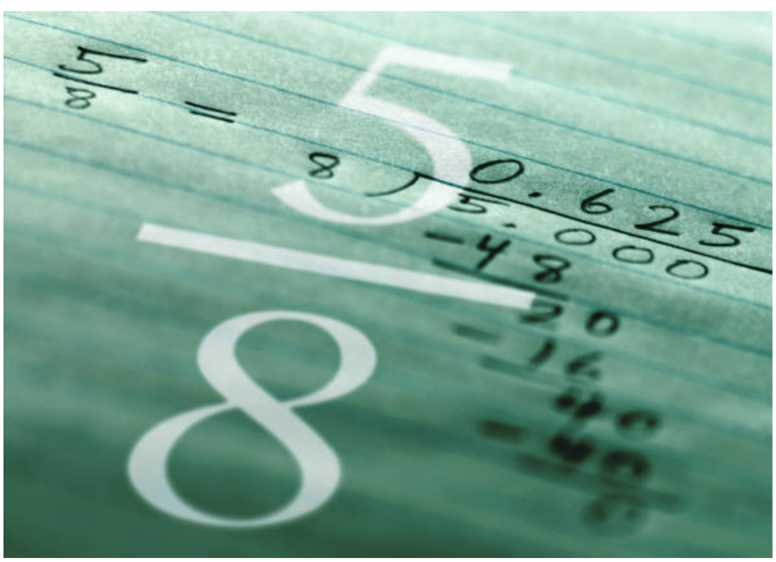


Here's another way to look at fractions. Imagine that each of these lines is a board and you want to cut it into halves, thirds, fourths, sixths, and fifths. Here is what those board fractions look like:

<table>
<tr><td colspan="60">one whole board</td></tr>
<tr><td colspan="30">1/2 board</td><td colspan="30">1/2 board</td></tr>
<tr><td colspan="15">1/4 board</td><td colspan="15">1/4 board</td><td colspan="15">1/4 board</td><td colspan="15">1/4 board</td></tr>
<tr><td colspan="20">1/3 board</td><td colspan="20">1/3 board</td><td colspan="20">1/3 board</td></tr>
<tr><td colspan="10">1/6 board</td><td colspan="10">1/6 board</td><td colspan="10">1/6 board</td><td colspan="10">1/6 board</td><td colspan="10">1/6 board</td><td colspan="10">1/6 board</td></tr>
<tr><td colspan="12">1/5 board</td><td colspan="12">1/5 board</td><td colspan="12">1/5 board</td><td colspan="12">1/5 board</td><td colspan="12">1/5 board</td></tr>
</table>

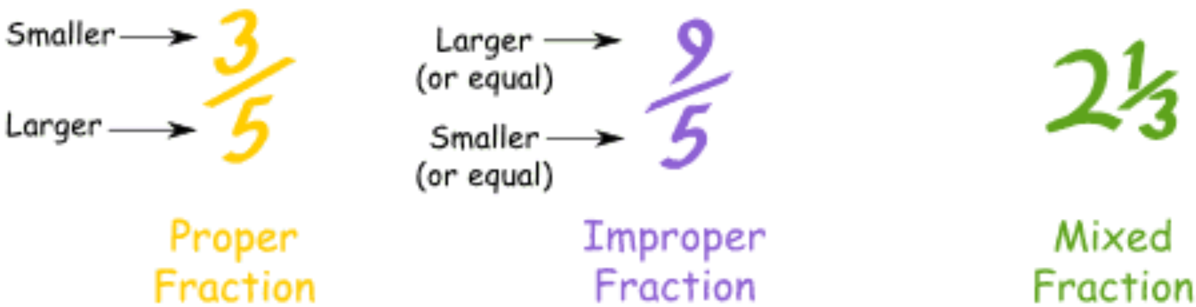
Notice that the 2 1/4th boards add up to a half board and that the 2 1/6 boards add up to a 1/3 board.

This means that 2/4=1/2 and 2/6=1/3.



Three Types of Fractions

There are three types of fractions:



Proper Fractions:	The numerator is less than the denominator. Examples: 1/3, 3/4, 2/7
Improper Fractions:	The numerator is greater than (or equal to) the denominator. Examples: 4/3, 11/4, 7/7
Mixed Fractions:	A whole number and proper fraction together. Examples: 1 1/3, 2 1/4, 16 2/5

Basic Fractions

Materials Needed

- square paper for each student
- scissors
- crayons

Directions

1. Pair students up.
2. Write the number $\frac{1}{2}$ on the board. Give each student a piece of square paper to illustrate $\frac{1}{2}$.
3. Ask student pairs to decide on how to represent $\frac{1}{2}$. (Often students have worked with fractions but are unaware of how to calculate or illustrate a fraction.)
4. Ask students to create both an illustration and an explanation to share with the class.
5. Accept or reject the ideas. Often students are able to show different ways to solve the same problem.

Solutions to the Fraction Problem of Showing $\frac{1}{2}$

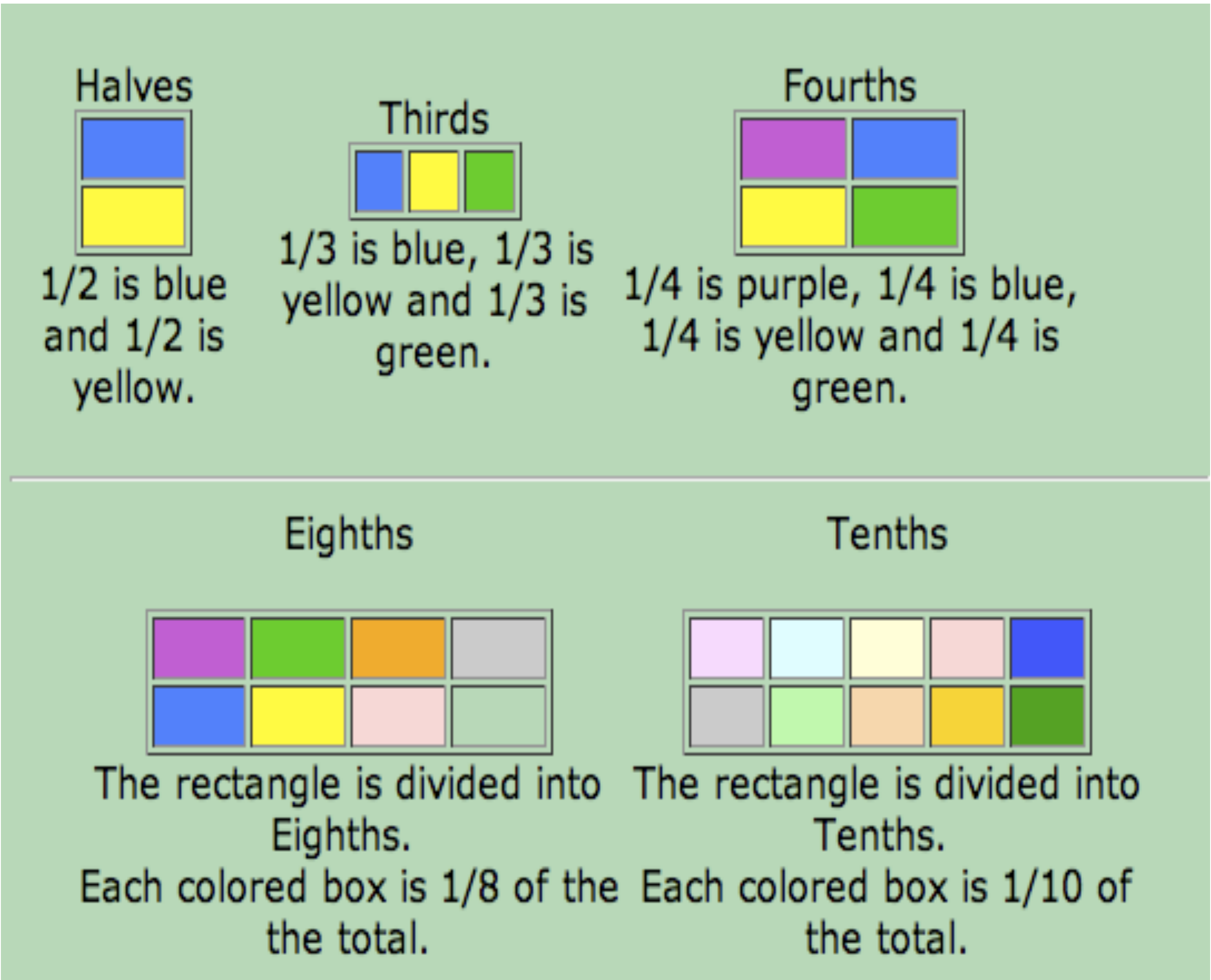
The paper is folded in half horizontally and one half of the paper is colored.
The paper is folded in half vertically and one half of the paper is colored.
The paper is folded in half horizontally and cut at the fold. One piece of the paper illustrates $\frac{1}{2}$.
The paper is folded in half vertically and cut at the fold. One piece of the paper illustrates $\frac{1}{2}$.

Students need to understand the concept that the piece of paper was a whole and it can be represented by two equal pieces. The two is the denominator and the part of the paper that is colored (one) represents the numerator.

Fraction Tips for Teachers

Illustrate one fraction only at the introduction of the topic.
Use a variety of mediums to illustrate the same fraction.
Use real life examples to represent a fraction.
Keep it simple.
Let students teach each other.
Incorporate the fraction concept into other areas of the curriculum.

Fourths, Eighths, and Tenths



Adding Fractions

Remember that given a fraction, such as 1/2, the top number is the numerator (namely 1) and the bottom number is the denominator (namely 2).

Example 1:

$$\frac{1}{5}+\frac{2}{5}=$$

Whenever you add fractions with *like* denominators, you add the **numerators** to create the answer numerator, and use the same **denominator** as the answer denominator.

$$\frac{1}{5}+\frac{2}{5}=\frac{1+2}{5}=\frac{3}{5}$$

Example 2:

$$\frac{3}{8}+\frac{1}{8}=$$

Whenever you add fractions with *like* denominators, you add the **numerators** to create the answer numerator, and use the same **denominator** as the answer denominator.

$$\frac{3}{8}+\frac{1}{8}=\frac{3+1}{8}=\frac{4}{8}=\frac{1}{2}$$

Comparing Fractions – Same Denominators

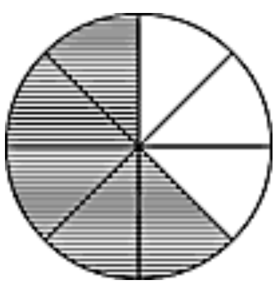
A Fraction consists of two numbers separated by a line.

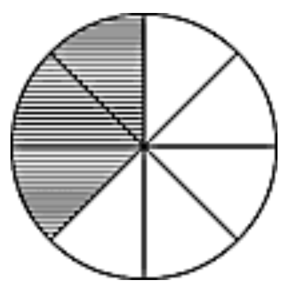
The top number (or numerator) tells how many fractional pieces there are. In the fraction 3/8, we have three pieces.

The denominator of a fraction tells how many pieces an object was divided into. The fraction 3/8 tells us that the whole object was divided into 8 pieces.

If the denominators of two fractions are the same, the fraction with the largest numerator is the larger fraction.

For example 5/8 is larger than 3/8 because all of the pieces are the same and five pieces are more than three pieces.





Comparing Fractions – Unlike Denominators

A Fraction consists of two numbers separated by a line.

The top number (or numerator) tells how many fractional pieces there are. The fraction 3/8 indicates that there are three pieces.

The denominator of a fraction tells how many pieces an object was divided into. The fraction 3/8 indicates that the whole object was divided into 8 pieces.

If the numerators of two fractions are the same, the fraction with the smaller denominator is the larger fraction.

For example 5/8 is larger than 5/12 because each fraction says there are five pieces but if an object is divided into 8 pieces, each piece will be larger than if the object were divided into 12 pieces. Therefore, five larger pieces are more than five smaller pieces.

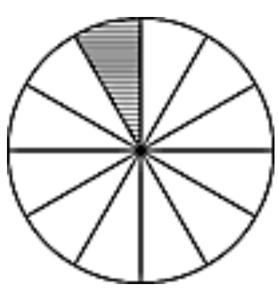
Comparing Fractions – Unlike Denominators

If two fractions have different numerators and denominators it is difficult to determine which fraction is larger. It is easier to determine which is larger if both fractions have the same denominator.

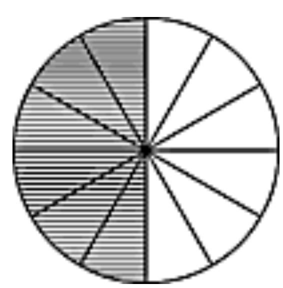
Multiply the numerator and denominator of one fraction by the same number so both fractions will have the same denominator. For example, if 5/12 and 1/3 are being compared, 1/3 should be multiplied by 4/4. It does not change the value of 1/3 to be multiplied by 4/4 (which is equal to 1) because any number multiplied by 1 is still the same number. After the multiplication (1/3 * 4/4 = 4/12), the comparison can be made between 5/12 and 4/12.

You may have to multiply both fractions by different numbers to produce the same denominator for both fractions. For example, if 2/3 and 3/4 are compared, we need to multiply 2/3 by 4/4 to give 8/12 and multiply 3/4 by 3/3 to give 9/12. The fraction 3/4, which is equal to 9/12, is larger than 2/3, which is equal to 8/12.

The fraction with the larger numerator is the larger fraction if the denominators are the same.



$$\frac{1}{12}$$



$$\frac{6}{12}$$

Equivalent Fractions

Equivalent Fractions have the same value – even though they may look different.

These fractions are really the same:

$$\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$$

Why are they the same? Because when you multiply or divide BOTH the top and bottom by the same number, the fraction keeps its value. The rule to remember is:

What you do to the top of the fraction
you must also do to the bottom of the fraction!

So, here is why those fractions are really the same:

1 x 2 = 2 2 x 2 = 4

$$\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$$

2 x 2 = 4 4 x 2 = 8

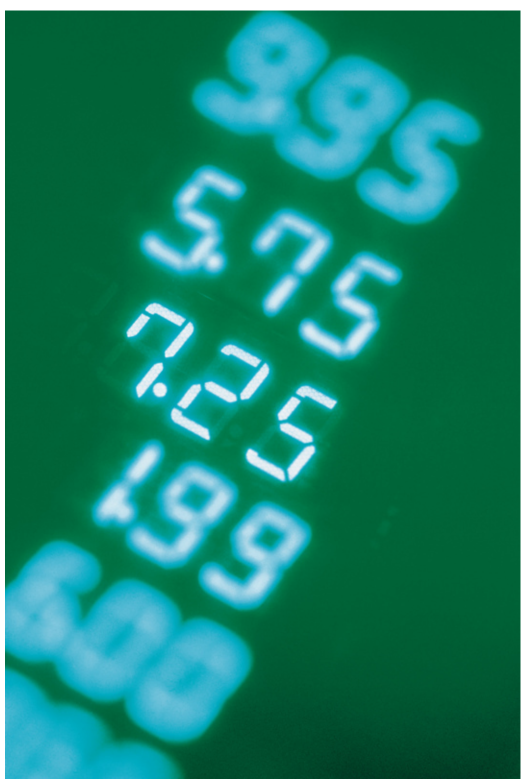
Comparing Decimals and Fractions

A decimal number and a fractional number can be compared. One number is either greater than, less than or equal to the other number.

When comparing fractional numbers to decimal numbers, convert the fraction to a decimal number by division and compare the decimal numbers.

If one decimal has a higher number on the left side of the decimal point then it is larger. If the numbers to the left of the decimal point are equal but one decimal has a higher number in the tenths place then it is larger and the decimal with fewer tenths is smaller. If the tenths are equal, compare the hundredths, then the thousandths, etc. until one decimal is larger or there are no more places to compare.

It is often easy to estimate the decimal from a fraction. If this estimated decimal is obviously much larger or smaller than the compared decimal – then it is not necessary to convert the fraction to a decimal.



Converting Improper Fractions to Mixed Numbers

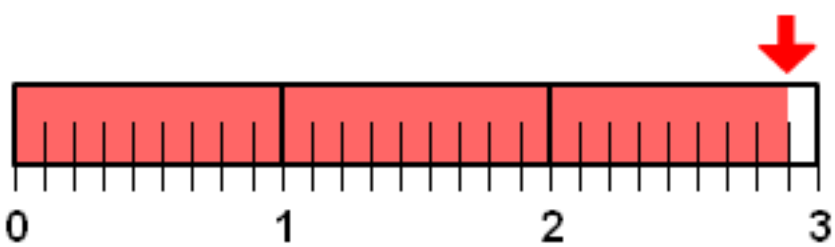
Fractions consist of two numbers. The top number is called the numerator. The bottom number is called the denominator.

$$\frac{\text{numerator}}{\text{denominator}}$$

An improper fraction is a fraction that has a numerator larger than or equal to its denominator. A proper fraction is a fraction with the numerator smaller than the denominator.

A mixed number consists of an integer followed by a proper fraction.

Example: The improper fraction 8/5 can be changed to the mixed number 1 3/5 by dividing the numerator (8) by the denominator (5). This gives a quotient of 1 and a remainder of 3. The remainder is placed over the divisor (5).



Fraction Form to Mixed Form

$$\frac{26}{9} = 2\,\frac{8}{9}$$

Changing Mixed Numbers to Improper Fractions

Fractions consist of two numbers. The top number is called the numerator. The bottom number is called the denominator.

$$\frac{\text{numerator}}{\text{denominator}}$$

An improper fraction is a fraction that has a numerator larger than or equal to its denominator. A proper fraction is a fraction with the numerator smaller than the denominator.

A mixed number consists of an integer followed by a proper fraction.

Example: The mixed number, 3 3/5, can be changed to an improper fraction by converting the integer portion to a fraction with the same denominator as the fractional portion and then adding the two fractions. In this case the integer portion (3) is converted to 15/5. The sum of the two fractions becomes 15/5 + 3/5 = 18/5.

The entire conversion is:

3 3/5 = 15/5 + 3/5 = 18/5

Converting Fractions to Percents

Do the following steps to convert a fraction to a percent: For example: Convert 4/5 to a percent.

- Divide the numerator of the fraction by the denominator (e.g. 4 ÷ 5=0.80)
- Multiply by 100 (Move the decimal point two places to the right) (e.g. 0.80*100 = 80)
- Round the answer to the desired precision.

- Follow the answer with the % sign (e.g. 80%)

Converting a Percent to a Fraction

Do the following steps to convert a percent to a fraction: For example: Convert 83% to a fraction.

Remove the Percent sign.
Make a fraction with the percent as the numerator and 100 as the denominator (e.g. 83/100).

Reduce the fraction if needed.

$$0.25 = \frac{25}{100} = \frac{1}{4}$$

Converting a Fraction to a Decimal

Do the following steps to convert a fraction to a decimal: For example: Convert 4/9 to a decimal.

- Divide the numerator of the fraction by the denominator (e.g. 4 ÷ 9=0.44444).
- Round the answer to the desired precision.

Or try this:
You might want to convert a number like 4 26/1000 into a decimal number.

- The 4 (whole things) simply stays as a 4, and then you look at the fraction part.
- Well, in this case, for the fraction part, we have 26/1000 - notice the three zeros at the bottom.
- Well, if we have three zeros at the bottom, then we *must* have three digits coming after the decimal point. So, the fraction part is simply .026.
- And, putting it all together, we have ... 4.026.
- Here are two more examples:
  - 14 6/100 = 14.06
  - 7 329/100000 = 7.00329

7.00329

Reciprocals of Fractions

The product of a number and its reciprocal equals 1.

The reciprocal of 4 is 1/4. The reciprocal of 2/3 is 3/2.

The reciprocal of 1 is 1.

The number 0 does not have a reciprocal because the product of any number and 0 equals 0.

$$\frac{1}{4} = \frac{4}{1}$$

Dividing Fractions by Whole Numbers

To Divide Fractions by Whole Numbers:

- Treat the integer as a fraction (i.e. place it over the denominator 1)
- Invert (i.e. turn over) the denominator fraction and multiply the fractions
- Multiply the numerators of the fractions
- Multiply the denominators of the fractions
- Place the product of the numerators over the product of the denominators
- Simplify the Fraction

Example: Divide 2/9 by 2

- The integer divisor (2) can be considered to be a fraction (2/1)
- Invert the denominator fraction and multiply (2/9 ÷ 2/1 = 2/9 * 1/2)
- Multiply the numerators (2*1=2)
- Multiply the denominators (9*2=18)
- Place the product of the numerators over the product of the denominators (2/18)
- Simplify the Fraction if possible (2/18 = 1/9)

The Easy Way

- After inverting, it is often simplest to "cancel" before doing the multiplication. Canceling is dividing one factor of the numerator and one factor of the denominator by the same number.
- For example: 2/9 ÷ 2 = 2/9 ÷ 2/1 = 2/9*1/2 = (2*1)/(9*2) = (1*1)/(9*1) = 1/9

Dividing Fractions by Fractions

To Divide Fractions:

- Invert (i.e. turn over) the denominator fraction and multiply the fractions.
- Multiply the numerators of the fractions.
- Multiply the denominators of the fractions.
- Place the product of the numerators over the product of the denominators.
- Simplify the fraction.

Example: Divide 2/9 and 3/12

- Invert the denominator fraction and multiply (2/9 ÷ 3/12 = 2/9 * 12/3).
- Multiply the numerators (2*12=24).
- Multiply the denominators (9*3=27).
- Place the product of the numerators over the product of the denominators (24/27).
- Simplify the fraction (24/27 = 8/9).

The Easy Way: After inverting, it is often simplest to "cancel" before doing the multiplication. Canceling is dividing one factor of the numerator and one factor of the denominator by the same number.

For example: 2/9 ÷ 3/12 = 2/9*12/3 = (2*12)/(9*3) = (2*4)/(3*3) = 8/9

Dividing Mixed Numbers

Mixed numbers consist of an integer followed by a fraction.

Dividing two mixed numbers:

- Convert each mixed number to an improper fraction.
- Invert the improper fraction that is the divisor.
- Multiply the two numerators together.
- Multiply the two denominators together.
- Convert the result back to a mixed number if it is an improper fraction.
- Simplify the mixed number.

Convert each mixed number to an improper fraction.	50/8 ÷ 32/9
Invert the improper fraction that is the divisor and multiply.	50/8 * 9/32
Multiply the two numerators together.	50 * 9 = 450
Multiply the two denominators together.	8 * 32 = 256
Convert the result back to a mixed number.	450/256 = 1 194/256
Simplify the mixed number.	1 97/128

Multiplication

To Multiply Fractions:

- Multiply the numerators of the fractions.
- Multiply the denominators of the fractions.
- Place the product of the numerators over the product of the denominators.
- Simplify the fraction.

Example: Multiply 2/9 and 3/12

- Multiply the numerators (2*3=6).
- Multiply the denominators (9*12=108).
- Place the product of the numerators over the product of the denominators (6/108).
- Simplify the fraction (6/108 = 1/18).

The Easy Way:

- It is often simplest to "cancel" before doing the multiplication. Canceling is dividing one factor of the numerator and one factor of the denominator by the same number.

For example: 2/9 * 3/12 = (2*3)/(9*12) = (1*3)/(9*6) = (1*1)/(3*6) = 1/18

Multiplying Fractions by Whole Numbers

Multiplying a fraction by an integer follows the same rules as multiplying two fractions.

- An integer can be considered to be a fraction with a denominator of 1.
- Therefore when a fraction is multiplied by an integer the numerator of the fraction is multiplied by the integer.
- The denominator is multiplied by 1, which does not change the denominator.

Problem Solving:

If there are 5 people in your family and you each have 1/16 of a cake, how much cake do you have all together? This problem involves multiplying a whole number by a fraction. Whole numbers and fractions are often put together. To solve problems like this, it is important to learn how to multiply a whole number by a fraction.

Multiplying whole numbers by fractions is almost exactly the same as multiplying a fraction by another fraction. This is because you can easily change whole numbers into fractions. To change a whole number into a fraction, you simply put the whole number on top and 1 on the bottom. You can do this because the whole number divided by one is the same as the whole number. For example, $\frac{6}{1} = 6$.

If you want to multiply $6 \times \frac{7}{3}$, first change 6 into a fraction. Write 6 on the top and 1 on the bottom to get $\frac{6}{1}$. Now multiply $\frac{6}{1} \times \frac{7}{3}$.

First, multiply the numbers on the top: 6 times 7 equals 42 (6 x 7 = 42). So 42 will go on the top of the answer. Next, multiply the numbers on the bottom: 1 times 3 equals 3 (1 x 3 = 3). The number on the bottom of the answer is 3. Put the two answers together in our fraction and see that $6 \times \frac{7}{3} = \frac{42}{3}$.

Multiplying Mixed Numbers

Mixed numbers consist of an integer followed by a fraction.

- Multiplying two mixed numbers:
- Convert each mixed number to an improper fraction.
- Multiply the two numerators together.
- Multiply the two denominators together.
- Convert the result back to a mixed number if it is an improper fraction.
- Simplify the mixed number.

Convert each mixed number to an improper fraction.	50/8 * 32/9
Multiply the two numerators together.	50 * 32 = 1600
Multiply the two denominators together.	8 * 9 = 72
Convert the result to a mixed number.	1600/72 = 22 16/72
Simplify the mixed number.	22 2/9

Prime and Composite Numbers

A prime number is a whole number that only has two factors, which are itself and one. A composite number has factors in addition to one and itself.

The numbers 0 and 1 are neither prime nor composite.

All even numbers are divisible by two and so all even numbers greater than two are composite numbers.

All numbers that end in five are divisible by five. Therefore all numbers that end with five and are greater than five are composite numbers.

The prime numbers between 2 and 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Factors

A number may be made by multiplying two or more other numbers together. The numbers that are multiplied together are called factors of the final number. All numbers have a factor of one since one multiplied by any number equals that number. All numbers can be divided by themselves to produce the number one. Therefore, we normally ignore one and the number itself as useful factors.

The number fifteen can be divided into two factors which are three and five.

The number twelve could be divided into two factors which are 6 and 2. Six could be divided into two further factors of 2 and 3. Therefore the factors of twelve are 2, 2, and 3.

If twelve was first divided into the factors 3 and 4, the four could be divided into factors of 2 and 2. Therefore the factors of twelve are still 2, 2, and 3.

There are several clues to help determine factors.

- Any even number has a factor of two
- Any number ending in 5 has a factor of five
- Any number above 0 that ends with 0 (such as 10, 30, 1200) has factors of two and five.

To determine factors see if one of the above rules applies (ends in 5, 0 or an even number). If none of the rules apply, there still may be factors of 3 or 7 or some other number.

3 x 4 = 12
3 and 4 are both factors of 12.
12 is a multiple of both 3 and 4

Greatest Common Factor

The Greatest Common Factor (GCF) is the largest number that is a common factor of two or more numbers.

How to find the greatest common factor:

- Determine if there is a common factor of the numbers. A common factor is a number that will divide into both numbers evenly. Two is a common factor of 4 and 14.
- Divide all of the numbers by this common factor.
- Repeat this process with the resulting numbers until there are no more common factors.
- Multiply all of the common factors together to find the Greatest Common Factor.

Prime factors of 18: 2 x 3 x 3

Prime factors of 24: 2 x 2 x 2 x 3

There is one 2 and one 3 in common.
The GCF is 2 X 3 = 6.

Least Common Denominator

The Least Common Denominator (LCD) is the Least Common Multiple of two or more denominators.

How to find the Least Common Denominator:

- Find the Greatest Common Factor of the denominators.
- Multiply the denominators together.
- Divide the product of the denominators by the Greatest Common Factor.

Example: Find the LCD of 2/9 and 3/12:

- Determine the Greatest Common Factor of 9 and 12, which is 3.
- Either multiply the denominators and divide by the GCF (9*12=108, 108/3=36)
- OR - Divide one of the denominators by the GCF and multiply the quotient times the other denominator (9/3=3, 3*12=36).

How to rename fractions and use the Least Common Denominator:

- Divide the LCD by one denominator.
- Multiply the numerator times this quotient.
- Repeat the process for the other fraction(s).

Example: Add 2/9 + 3/12:

- LCD is 36.
- First fraction (2/9): 36/9 = 4, 4*2 = 8, first fraction is renamed as 8/36.
- Second fraction (3/12): 36/12 = 3, 3*3 = 9, second fraction is renamed as 9/36.
- It is possible to add or subtract fractions that have the same denominator 8/36 + 9/36 = 17/36.

Least Common Multiple

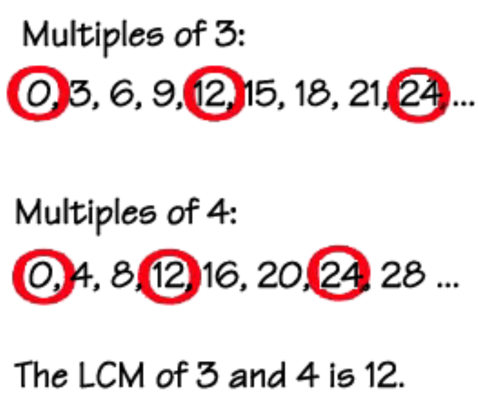
The Least Common Multiple (LCM) is the smallest number that two or more numbers will divide into evenly.

How to find the Least Common Multiple of two numbers:

- Find the Greatest Common Factor (GCF) of the numbers.
- Multiply the numbers together.
- Divide the product of the numbers by the GCF.

Example: Find the LCM of 15 and 12

- Determine the Greatest Common Factor of 15 and 12 which is 3.
- Either multiply the numbers and divide by the GCF (15*12=180, 180/3=60).
- OR - Divide one of the numbers by the GCF and multiply the answer times the other number (15/3=5, 5*12=60).



Simplifying Fractions

Fractions may have numerators and denominators that are composite numbers (numbers that have more factors than 1 and themselves).

How to simplify a fraction:

- Find a common factor of the numerator and denominator. A common factor is a number that will divide into both numbers evenly. Two is a common factor of 4 and 14.
- Divide both the numerator and denominator by the common factor.
- Repeat this process until there are no more common factors.
- The fraction is simplified when no more common factors exist.

Another method to simplify a fraction:

- Find the Greatest Common Factor (GCF) of the numerator and denominator. Divide the numerator and the denominator by the GCF.

Reduce $\frac{30}{45}$ to lowest terms.

step 1 step 2 step 3

$$\frac{30}{45} = \frac{2 \times 3 \times 5}{3 \times 3 \times 5} = \frac{2 \times \cancel{3} \times \cancel{5}}{3 \times \cancel{3} \times \cancel{5}} = \frac{2}{3}$$

Reduce $\frac{6}{8}$ to lowest terms.

step 1 step 2 step 3

$$\frac{6}{8} = \frac{2 \times 3}{2 \times 2 \times 2} = \frac{\cancel{2} \times 3}{\cancel{2} \times 2 \times 2} = \frac{3}{4}$$

Divisibility Rules

Dividing by 2

- All even numbers are divisible by 2. I.e., all numbers ending in 0,2,4,6 or 8.

Dividing by 3

- Add up all the digits in the number.
- Find out what the sum is. If the sum is divisible by 3, so is the number.
- For example: 12123 (1+2+1+2+3=9) 9 is divisible by 3, therefore 12123 is too!

Dividing by 4

- Are the last two digits in your number divisible by 4?
- If so, the number is too!
- For example: 358912 ends in 12, which is divisible by 4, thus so is 358912.

Dividing by 5

- Numbers ending in a 5 or a 0 are always divisible by 5.

Dividing by 6

- If the Number is divisible by 2 and 3, it is divisible by 6 also.

Dividing by 7 (2 Tests)

- Take the last digit in a number.
- Double and subtract the last digit in your number from the rest of the digits.
- Repeat the process for larger numbers.

Example: 357 (Double the 7 to get 14. Subtract 14 from 35 to get 21, which is divisible by 7, and we can now say that 357 is divisible by 7.

NEXT TEST

- Take the number and multiply each digit beginning on the right hand side (ones) by 1, 3, 2, 6, 4, 5. Repeat this sequence as necessary.
- Add the products.
- If the sum is divisible by 7 - so is your number.
- Example: Is 2016 divisible by 7?
- 6(1) + 1(3) + 0(2) + 2(6) = 21.
- 21 is divisible by 7, and we can now say that 2016 is also divisible by 7.

Dividing by 8

- This one's not as easy. If the last 3 digits are divisible by 8, so is the entire number.
- Example: 6008 - The last 3 digits are divisible by 8, therefore, so is 6008.

Dividing by 9

- Almost the same rule and dividing by 3. Add up all the digits in the number.
- Find out what the sum is. If the sum is divisible by 9, so is the number.
- For example: 43785 (4+3+7+8+5=27) 27 is divisible by 9, therefore 43785 is too!

Dividing by 10

- If the number ends in a 0, it is divisible by 10.

Identifying Equivalent Decimals to Fractions

Decimals are a type of fractional number. The decimal 0.5 represents the fraction 5/10. The decimal 0.25 represents the fraction 25/100. Decimal fractions always have a denominator based on a power of 10.

We know that 5/10 is equivalent to 1/2 since 1/2 times 5/5 is 5/10. Therefore, the decimal 0.5 is equivalent to 1/2 or 2/4, etc.

Some common Equivalent Decimals and Fractions:

- 0.1 and 1/10
- 0.2 and 1/5
- 0.5 and 1/2
- 0.25 and 1/4
- 0.50 and 1/2
- 0.75 and 3/4
- 1.0 and 1/1 or 2/2 or 1

Subtraction Fractions – Same Denominators

Fractions consist of two numbers. The top number is called the numerator. The bottom number is called the denominator.

$$\frac{\text{numerator}}{\text{denominator}}$$

To subtract two fractions with the same denominator, subtract the numerators and place that difference over the common denominator.

$$\frac{33}{52} - \frac{13}{52} = \frac{20}{52}$$

Subtracting Fractions – Different Denominators

To Subtract Fractions with different denominators:

- Find the Lowest Common Denominator (LCD) of the fractions.
- Rename the fractions to have the LCD.
- Subtract the numerators of the fractions.
- The difference will be the numerator and the LCD will be the denominator of the answer.
- Simplify the fraction.

Example: Find the difference between 3/12 and 2/9.

- Determine the Greatest Common Factor of 12 and 9, which is 3
- Either multiply the numbers and divide by the GCF (9*12=108, 108/3=36)
- OR - Divide one of the numbers by the GCF and multiply the answer times the other number (12/3=4, 9*4=36).
- Rename the fractions to use the Lowest Common Denominator (3/12=9/36, 2/9=8/36).
- The result is 9/36 - 8/36.
- Subtract the numerators and put the difference over the LCD = 1/36.
- Simplify the fraction if possible. In this case it is not possible.

Subtracting Mixed Numbers

A mixed numbers consists of an integer followed by a fraction.

How to subtract mixed numbers having the same denominator:

- Make the first numerator larger than the second if it is not.
- Subtract the second numerator from the first.
- Place that difference over the common denominator.
- Subtract the integer portions of the two mixed numbers.
- State the answer

Example: 5 1/3 - 3 2/3 =

Make the first numerator larger than the second	5 1/3 = 4 4/3
Subtract the fractional parts of the mixed numbers	4/3 - 2/3 = 2/3
Subtract the integer portions of the mixed numbers	4 - 3 = 1
State the final answer:	1 2/3

Games

Paper Folding Fraction Game

Start with a blank piece of blank paper. Have student fold it in half, along either direction, and then quickly color one half of it. Then we continue folding and counting the colored sections: 2/4 4/8 8/16 ... but hasn't the amount of the paper we colored stayed the same? Then try thirds, sixths ...

Candy Fractions - Fraction Paper Game

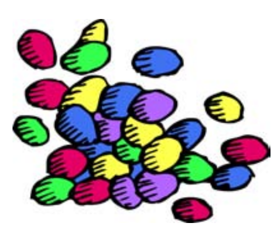
by http://hannahmeans.bizland.com (A New Teacher's Survival Guide)

To make sure that children really understand fractions, I get their attention by bringing out Hershey Bars. First we look at a bar and determine how many sections it is divided into. We talk about how each section is 1/12 of the whole bar. Then I pass out rectangles of brown construction paper. They divide the paper as the candy is divided and mark each section as 1/12.

Then I break the candy bar in half. We talk about all the different ways that we could divide the candy bar in halves. The children cut their paper candy bars in two. Then we talk about what is in each half. We cut one of the halves in half and I write on the board all the statements that the children can make about their "candy bars" i.e.: "There are six twelfths in each half," "there are two halves in a whole," "there are three twelfths in a fourth," etc.

After the children are familiar with the basics of Hershey Bar fractions, I introduce M & M fractions. I get a regular size bag of peanut M & M's. I open them and we try to divide them evenly. (If I am lucky, and there are an odd number of candies in the bag, I correct the problem by eating one.) Then I give the children a sheet of paper with a bag drawn on it. They draw the correct number of M & M's in the bag with colored crayons. We divide the M&M's into two piles. They cut their picture of the M&M's in two. Then we follow the same procedure that we did with the Hershey Bar fractions.

Of course, if you have a small class, you can use the actual candy bars and M & M bags instead of the paper counterparts.



Make a pizza or pie fraction game. For a two-player game, print out or draw pictures of at least seven whole pizzas or pies. Leave two of the printouts whole for game boards. Segregate the remaining five printouts into fractional parts and label them. Use the fractions your students are currently working on. For example, mark one pizza into halves, one into thirds, one into fourths, and so on. Cut these pizzas or pies into their fractional parts. Laminate, if possible, for durability.

To play the pizza fraction game, give each player a whole pizza or pie game board. Place the fractional pieces inside an opaque envelope or bag to be randomly selected by the players. Each player, in turn, takes a piece of the pizza from the envelope and places it on his whole pizza game board. The winner is the one who can cover the entire pizza first. If both players go over a whole, either disregard that turn and continue play or choose the winner according to who came closest to a whole.

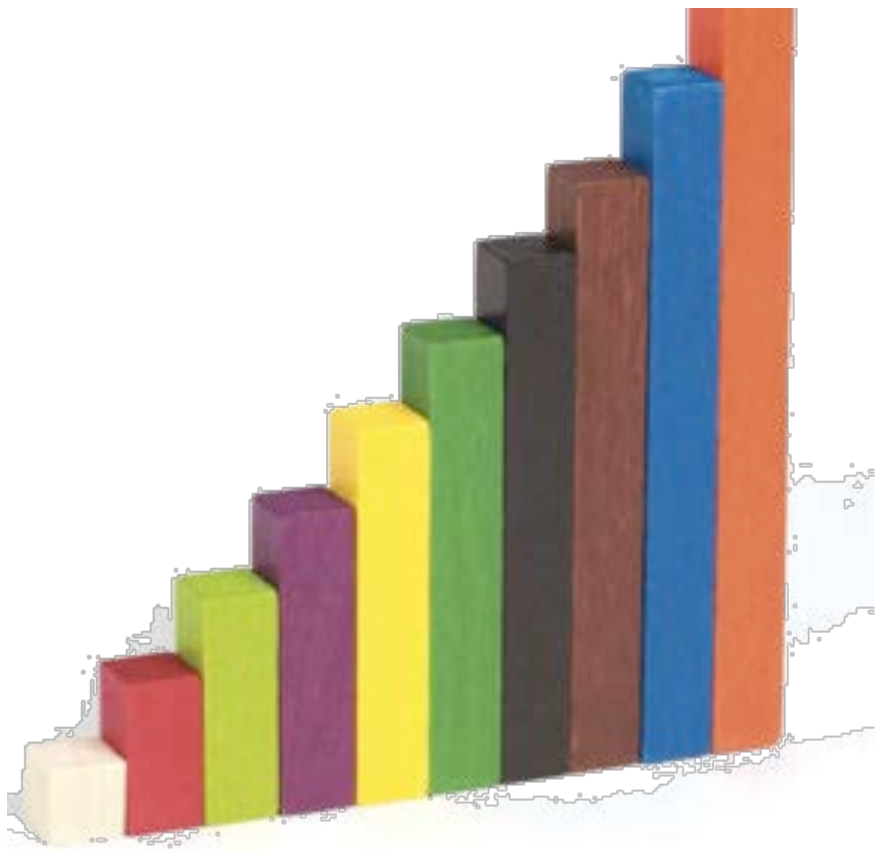
Make a fraction card game. Using plain paper index cards, create a deck of cards that has fractions in both number and picture form. Make the entire set in numbers and pictures for each fractional denominator. For example, make numbers and pictures for 1/3, 2/3 and 3/3. Do this for all fractions you will use. Use easily recognizable fractions, such as halves, thirds, fourths and eights.

Later, add more fractions to the deck or create more difficult decks with other fractions.

Play the fraction card game in a variety of ways. Play a memory-type game with the cards by turning them face down, then taking turns picking them up, trying to make picture or number matches. For example, the picture of 1/2 could match the number card 1/2 or either the picture or number card for 2/4. Play other traditional kids' card games with the fraction deck, including War (see who has the larger fraction) or Go Fish (see if the other player has a matching fraction in her hand). Raise the difficulty level of any of these games by introducing concepts like equivalent fractions and by adding fractions.

Make a bar fraction game. Use sentence strips, adding-machine tape or any long, thin pieces of paper that are all the same length. Leave one piece whole and cut the others into fractional parts such as halves, fourths, thirds, sixths and eighths. Each player needs a set of these pieces. Create these yourself or have kids make them as part of their math practice time. Before playing games with them, have kids lay them down in decreasing order, one set under the other, from whole to eighths. This provides a visual picture of the different fractional-equivalent pieces.

Play the fraction bar game in the same way as the pizza or pie fraction game. Use the whole strip of paper as the game board. Place the fraction pieces in an opaque envelope or bag. Continue this game in the same manner as the pizza game above.



Chocolate fractions

Draw equal fractions that you can see using a Kit Kat out of the wrapper.

Draw a whole bar.

Draw half a bar and write it as a fraction.

Draw how many halves = a whole bar and write it as a fraction.

Draw a quarter of a bar and write it as a fraction.

Draw how many quarters = a whole bar and write these as fractions.

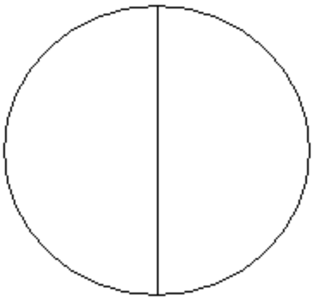
Draw how many quarters = half a bar and write these as a fraction.

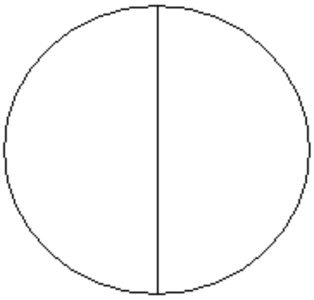
Equivalent Pizza Fractions

Halve the pizza slices to create equivalent fractions.

2 people share a pizza. They have 1 slice each. What fraction is each slice?

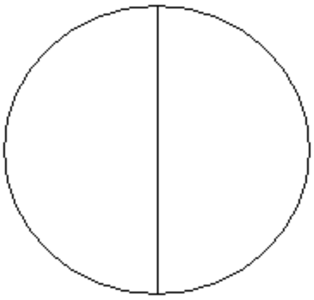
____





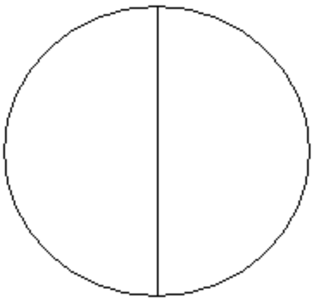
Each person wants two different toppings. Divide the pizza below correctly. What fraction is each slice?

____



Two more friends now join. Alter the pizza below so that each person has two slices of pizza each. What fraction is each slice?

____



The four friends decide to order a family size pizza so that each person can have four pieces. Draw how many slices the new pizza will have. What fraction is each slice?

____

<u>Can you answer these questions on equivalent fractions?</u>

1. How many ⌒ in ▭ ?
2. How many eighths in ▭?
3. How many eighths in ⌒?
4. How many eighths in ▭
5. Hoe many sixteenths in one eighth?
6. How many sixteenths in three eighths?
7. How many sixteenths in ▭?
8. How many sixteenths in ⌒?
9. How many ⌒s is twelve sixteenths?
10. write the fractions that are equivalent to:

▭ __________

⌒ __________

▭ __________

Write a rule for what happens to each pizza slice fraction each time the pizza slice is halved:

____

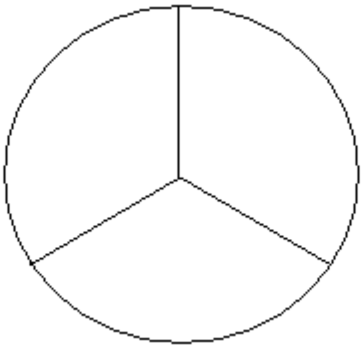
____

____

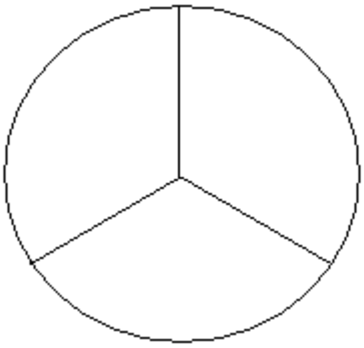
Equivalent Cheesecake Fractions

Halve the cheesecake slices to create equivalent fractions.

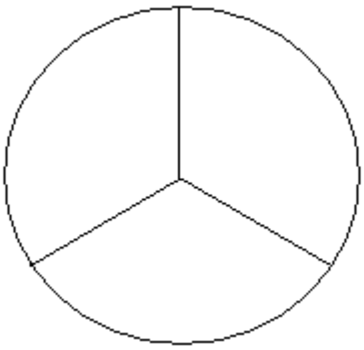
Miss Crawford shares her cake with Mrs Murphy and Mrs Croasdale. What fraction is each slice?



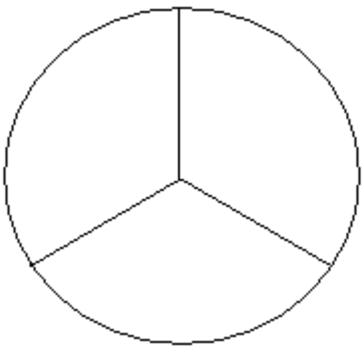
Halve each of the slices in the cheesecakes below to create further equivalent fractions (clue: the next ones are sixths!).



Fractions:__________



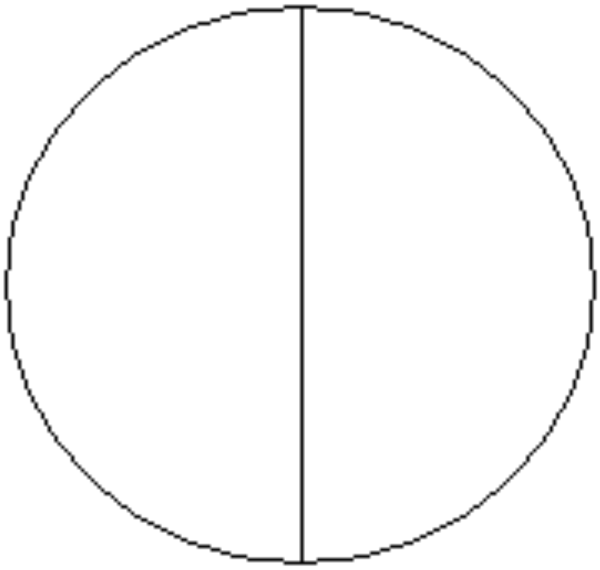
Fractions:__________



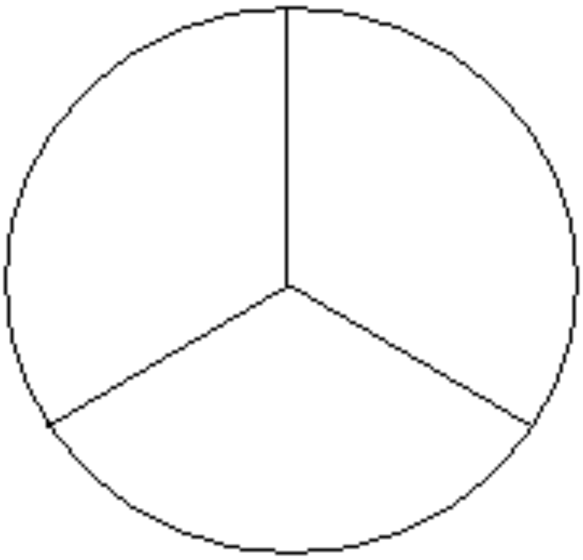
Fractions:__________

In the pizza below write fractions <u>equivalent</u> to ▭.

Remember to use **<u>both</u>** sides to the worksheet to help you. (Clue: look at any even numbered fractions).



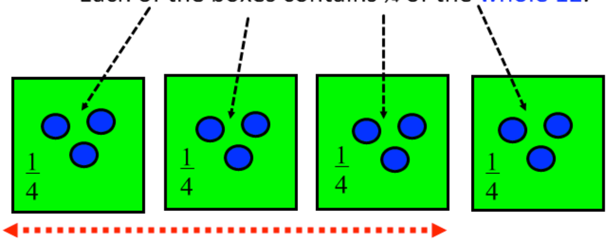
In the cheesecake below write in any fractions <u>equivalent</u> to 1/3. Remember to use both sides of the work sheet to help you!



Finding Fractions of Numbers and Shapes.

L.o. To use division to find fractions of numbers and shapes

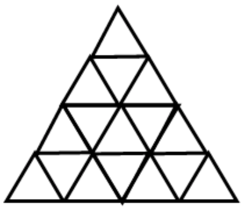
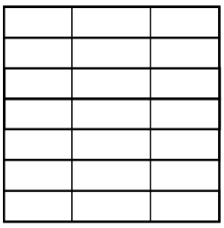
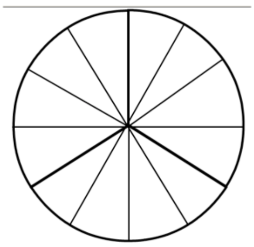
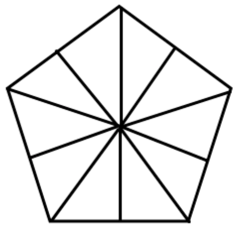
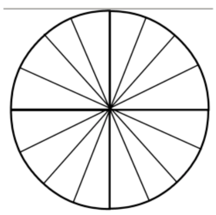
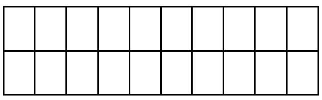
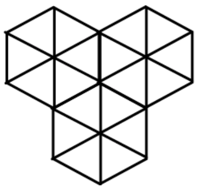
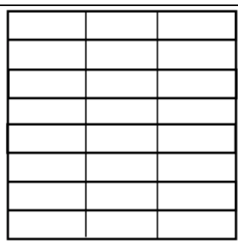
To find a fraction of a number you divide by the bottom number and multiply by the top number. This is because the denominator tells us how much to break the whole into and the numerator tells us how much of the whole we are talking about.

To find ¾ of 12: **12 ÷ 4 = 3** **(Divide the whole by the denominator)** **3 x 3 = 9** **(Multiply the answer by the numerator)** **So ¾ of 12 = 9**	The whole 12 has be shared out between 4 Each of the boxes contains ¼ of the whole 12.  We are asked to find ¾ of the whole so we must look at 3 of the 4 boxes. ¾ of 12 is 9

5. ¼ of 32 = ___

10. ¼ of 36 = ___

Now shade the correct amount of these shapes.

 ⅛	 ⅓		
 1/10	 ½	 1/9	

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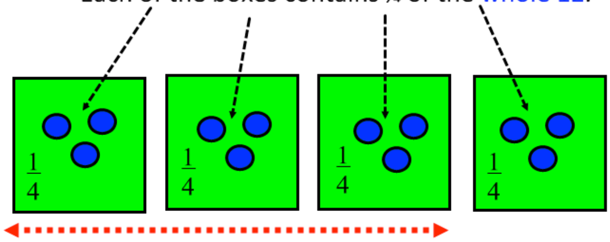
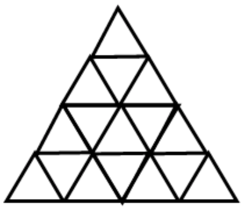
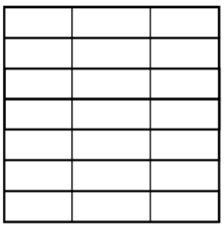
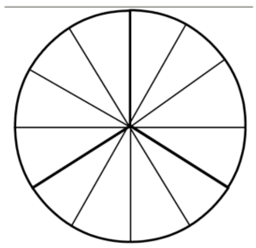
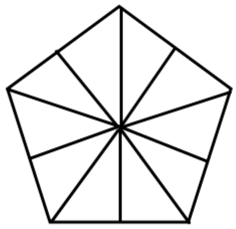
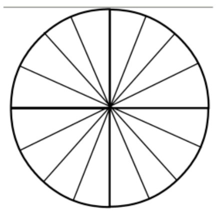
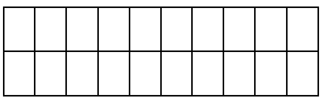
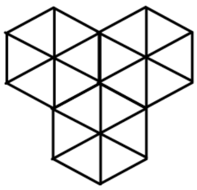
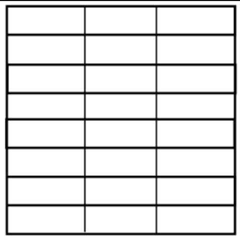
To find ¾ of 12: **12 ÷ 4 = 3** **(Divide the whole by the denominator)** **3 x 3 = 9** **(Multiply the answer by the numerator)** **So ¾ of 12 = 9**	The whole 12 has be shared out between 4 Each of the boxes contains ¼ of the whole 12.  We are asked to find ¾ of the whole so we must look at 3 of the 4 boxes. ¾ of 12 is 9

5. ¼ of 32 = ___ 10. ⅚ of 36 = ___ 15. ⅜ of 48 = ___

Now shade the correct amount of these shapes.

 ⅛	 ⅓	 ☐	 ☐
 ¾	 **8/10**	 **2/9**	 ☐

Finding Fractions of Numbers and Shapes.

L.o. To use division to find fractions of numbers and shapes

To find a fraction of a number you divide by the bottom number and multiply by the top number. This is because the denominator tells us how much to break the whole into and the numerator tells us how much of the whole we are talking about.

To find ¾ of 12:

12 ÷ 4 = 3

(Divide the whole by the denominator)

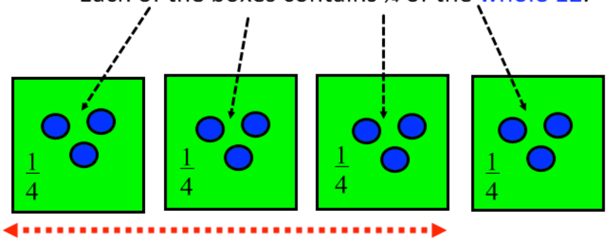
3 x 3 = 9

(Multiply the answer by the numerator)

So ¾ of 12 = 9

The whole 12 has be shared out between 4

Each of the boxes contains ¼ of the whole 12.



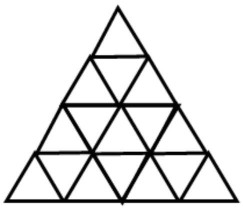
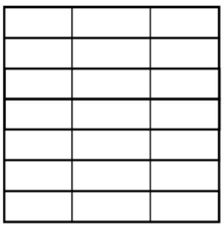
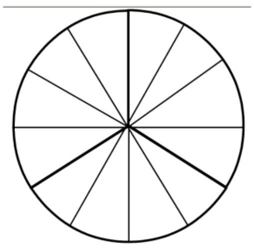
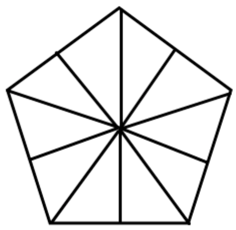
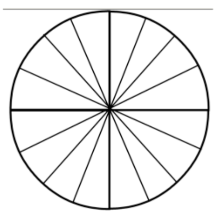
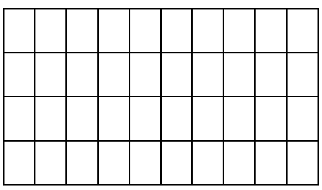
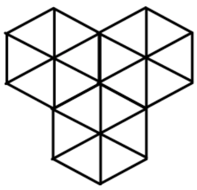
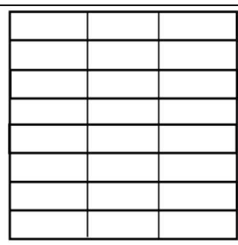
We are asked to find ¾ of the whole so we must look at 3 of the 4 boxes. ¾ of 12 is 9

5. ⅞ of 32 = ___

10. ⅚ of 36 = ___

15. ⅜ of 72 = ___

Now shade the correct amount of these shapes.

 3/8	 3/7		
 ¾	 8/10	 2/9	 5/12

Fraction Feud

Skills: Comparing fractions

Materials: 1 deck of “Fraction Feud” Cards

Number of Players: Two or more

Ages: 8+

Playing the Game

The object of the game is to win the most “battles” and to have the most cards at the end of the game!

1. Shuffle the Fraction Feud cards and deal them out so that each player has the same number of cards. Each player must keep their cards in a pile face down, and players cannot look at their cards.

2. Each player turns their top card face up and puts it on the table. Players compare their fractions. The player with the greatest fraction takes all of the cards in play and adds them to the bottom of their pile (face down).

3. Play continues in this manner, with each player placing their next card in play and so on.

4. If there is a tie and the two highest fractions in play are equivalent (equal), then there is a feud! The tied cards stay on the table and both players place the next card from their pile face-down and then another face-up. The new face-up fractions are compared, and the winner takes all of the cards in play (including those that are face down). If the new fractions are equivalent, then the feud continues in the same manner. The feud goes on as long as the cards in play continue to be equal. As soon as they are different, the player with the greater fraction wins all the cards in the feud.

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$
$\frac{2}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{2}{5}$
$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{6}$	$\frac{2}{6}$

$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{1}{8}$
$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$
$\frac{6}{8}$	$\frac{7}{8}$	$\frac{1}{9}$	$\frac{2}{9}$

$$\frac{3}{9}$$ $$\frac{4}{9}$$ $$\frac{5}{9}$$ $$\frac{6}{9}$$

$$\frac{7}{9}$$ $$\frac{8}{9}$$ $$\frac{1}{10}$$ $$\frac{2}{10}$$

$$\frac{3}{10}$$ $$\frac{4}{10}$$ $$\frac{5}{10}$$ $$\frac{6}{10}$$

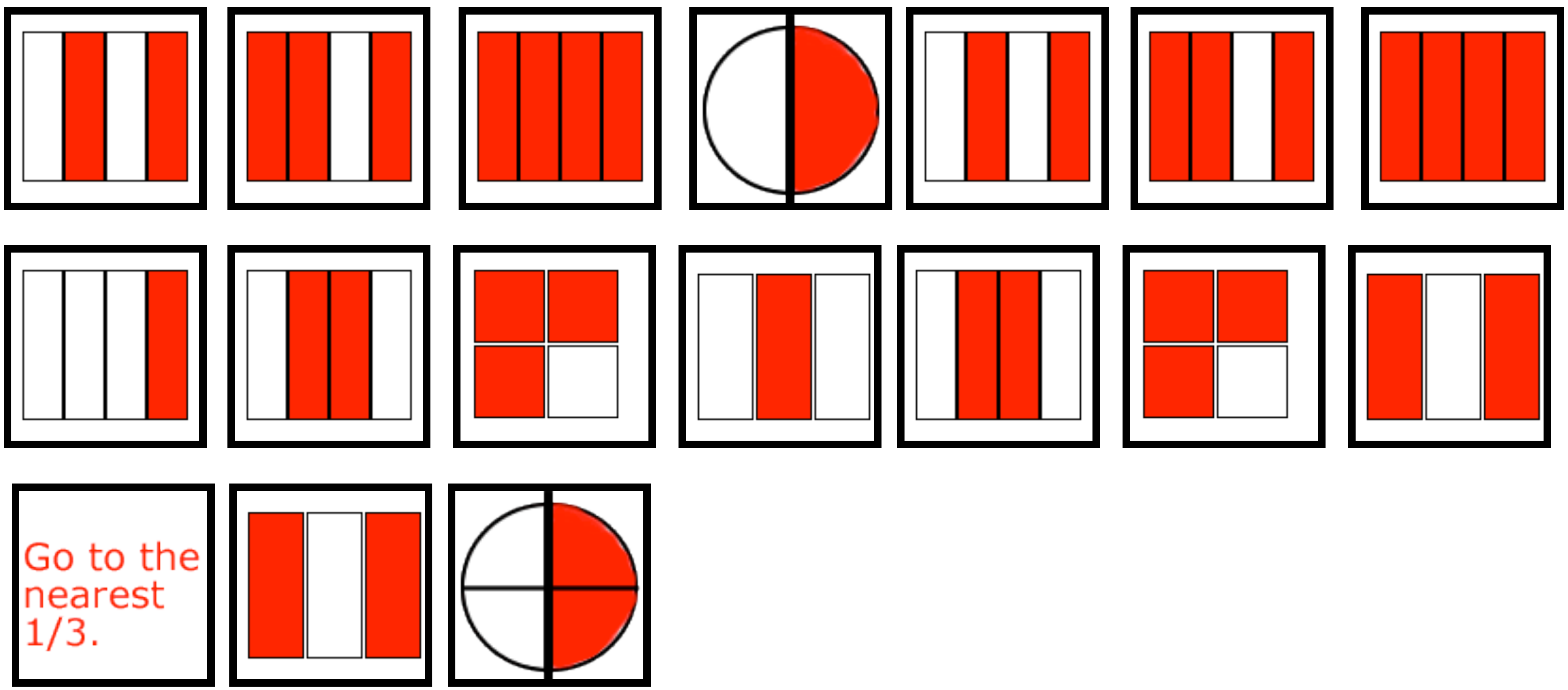
$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{1}{12}$
$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$
$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$

$\frac{10}{12}$	$\frac{11}{12}$	$\frac{10}{12}$	$\frac{1}{50}$
$\frac{10}{50}$	$\frac{25}{50}$	$\frac{40}{50}$	$\frac{1}{1}$
0	0	1	1


START
1/2
1/3
2/4
1/3
1/4
2/3
FINISH
1/4
3/4
READ THE DIRECTIONS
2/4
1/3
1/4
1/2
1
1
2/4
2/3
1
1/3
1/3
2/3
1
1/2
1/2
1/2

Instructions: Cut the cards below apart. Face down, spread them out for players to pick from when their turn comes up.

- Shortest player goes first. They select a card.
- They move their marker onto the square on the game board that is the same as what they have selected on the card.   (i.e. you roll a picture of one-half, you go to the first square with one-half (1/2) on it).
- You can decide how to interpret the fraction. But you must say 2/3 **red** or 1/3 **white** so that the other player knows how you are looking at the picture of the fraction. Use this to your advantage to move further along on the game board.
- First one to the finish line wins!
- Once a card has been used it is returned to the pile face down.



The Fraction Game

With this math fractions game, your kids can learn to add fractions the fun way. First, print out the fraction cards - preferably onto cardboard, and in color if you can. There are actually **three types** of cards in the pack.

- **Blue cards** with **fractions** having denominators 2, 3, 4, 5, 6, 10, 12 and 15.
- **Red ards**, with all the same fractions.
- "Wild" cards, in both red and blue.

If you leave out the wild cards, you'll have a set of 98 cards. You could use them to play games like **Snap!** or **Memory!** Because the pack contains cards like the one shown here (equivalent to one half, but written as two quarters), this will give your students good practice reducing fractions. Just imagine, in a game of Snap, as they try to figure out - quicker than their opponent - whether or not 4/10 is the same as 6/15!

Here is a more complex game (still simple enough) that will help teach students how to **add and subtract fractions,** too!

The goal...

- To make sets of cards which add up to 1.
- For example, 1/2 + 2/4 is a set. So is 2/15 + 1/2 + 1/6 + 2/10 a set.

At the start...

- Give each player a piece of paper and a pencil.
- Shuffle the cards, and deal 6 cards to each player.
- Place the rest of the cards **face down** on the table. This is the **draw pile**

On each player's turn...

- The player draws two cards. These two cards can be both from the draw pile, **or** both from the *discard pile* (see below), or one from each.
- If a player has a set of cards that adds up to 1, they should take the set out of their hand, and place it face up in front of them. At the end of the round, they'll get points for each set they make.
- The player can use their paper and pencil to work out sums if he or she likes. However, calculators and outside help are forbidden!
- If a player did not make a set, the player must discard one card face-up onto the **discard pile** (next to the draw pile).

The end of a round...

- When a player finishes all his or her cards, the round is over.
- All the other players give their cards to that person (but they keep the 'sets' they have already laid on the table).
- Points are awarded as follows:
  - Each card is worth the sum of the numerator and the denominator. So the card with 3/5 would be worth 8 points, and 2/10 would be worth 12 points.
  - 'Wild' cards are worth 1 point each.
- The round can also end if there are no more cards to draw. Then, each player just gets the points for the 'sets' they have laid out in front of them.

Wild Cards...

- The **X** of the wild card can count as any positive whole number, but not equal to the denominator.
- So X/15 could be 3/15, or 8/15, or 2/15.
- However, it can't be (2.5)/15 or 15/15, or 0/15.
- Remember, at the end of the round, wild cards are only worth 1 point!

For Example...

- Just suppose...
  - Larry has the cards 1/2, 1/5 and 2/15
  - Harry has the cards 3/4, 1/6 and 1/10
  - Carrie has the cards 8/10, 1/3 and 1/4.
- and it's Larry's turn.
  - Suppose he draws 2/3 and 7/12.
  - He thinks and thinks, but can't see any sets. (I suppose he needs more fractions practice.) Anyway, he discards his 1/5.

- So, at the end of Larry's turn,
  - Larry has the cards 1/2, 2/15, 2/3 and 7/12.
  - Harry still has the cards 3/4, 1/6 and 1/10,
  - Carrie still 8/10, 1/3 and 1/4, and
  - the card 1/5 is at the top of the discard pile, and
  - it's Harry's turn.
- Now, suppose Harry draws 3/12 and 7/15 from the draw pile.
  - After some thought, he realises that 3/4 + 3/12 = 1, so he lays them on the table to make a set.
  - He doesn't have to discard a card since he made a set. He decides not to (although if he had wanted to, he could).
- So, after Harry's turn,
  - Larry has the cards 1/2, 2/15, 2/3 and 7/12.
  - Harry has two cards in his hand, 1/10 and 1/6. He also has a set (3/4 and 3/12) on the table.
  - Carrie still has 8/10, 1/3 and 1/4.
  - The discard pile is still showing 1/5.
- Now, it's Carrie's turn.
  - She realises that she can use the 1/5 that Larry threw away, so she takes it.
  - She also draws - lucky her - a wild X/12 from the draw pile!
  - So she makes *two* sets... 8/10 + 1/5, and 1/3 + 1/4 + X/12 (with the X being 5). And she's finished all her cards!
- So, the round is over.
  - Poor Larry gives *all* his cards to Carrie.
  - Harry gives Carrie the two cards in his hand, but keeps his set (3/4 and 3/12).
  - Carrie has her two sets, and all of the cards Harry and Larry gave her. That is, 1/2, 2/15, 2/3 and 7/12 from Larry, 1/10 and 1/6 from Harry, and 8/10, 1/5, 1/3, 1/4 and X/12 from her own two sets.
  - Carrie gets a huge number of points - (3 + 17 + 5 + 19) + (11 + 7) + (18 + 6 + 4 + 5 + 1), a total of 96 points!
  - Harry gets some consolation, he gets 7 + 15 = 22 points from his set of two cards.
  - Poor Larry gets 0 this round. Maybe he'll have better luck next round?

You might like to vary the rules. Feel free! Here are a few suggestions.

- In the rules above, sets are formed by addition of fractions only. You could also allow subtraction using the red cards. So a blue 2/3, a blue 1/2, and a blue 1/6 could not form a set, but So a blue 2/3, a blue 1/2, and a red 1/6 could. So could a blue 5/6 and a red 1/6 - that's right, the red cards can be used for addition *or* subtraction.

- Instead of allowing each player his or her own piece of paper for scrap paper, everyone shares one piece of paper... Yes, that's right, they can see each other's figuring!

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$

$\frac{4}{5}$	$\frac{1}{6}$	$\frac{2}{6}$
$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$
$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$

$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$
$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$
$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$

$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$
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$\frac{10}{12}$	$\frac{11}{12}$	$\frac{1}{15}$

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$\frac{x}{10}$ $$\frac{X}{10}$$ $\frac{x}{10}$	$\frac{x}{12}$ $$\frac{X}{12}$$ $\frac{x}{12}$	$\frac{x}{15}$ $$\frac{X}{15}$$ $\frac{x}{15}$

Whole

1/1

Half

1/2

Half

1/2

Quarter 1/4	Quarter 1/4	Quarter 1/4	Quarter 1/4

Eighth	Eighth	Eighth	Eighth	Eighth	Eighth	Eighth	Eighth
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

Trade Books About Fractions

Fraction Action, by Loreen Leedy

Funny & Fabulous Fraction Stories, by Dan Greenberg and Jared Lee

Painless Fractions, by Alyece Cummings

Apple Fractions, by Jerry Pallotta and Rob Bolster

Fraction Fun, by David A. Adler and Nancy Tobin

Full House: An Invitation to Fractions, by Dayle Ann Dodds and Abby Carter

The Hershey's Milk Chocolate Bar Fractions Book, by Jerry Pallotta and Robert C. Bolster

Piece = Part = Portion, by Scott Gifford and Shmuel Thaler

If You Were a Fraction, by Speed Shaskan, Trisha, Carabelli, and Francesca

A Fraction of the Whole, by Steve Toltz

Eating Fractions, by Bruce McMillan