

Our World as Math

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Introduction

The purpose for creating this unit is to share and encourage educators and students that learning takes place when there is shared experience and dialogue. The work submitted is part of a larger unit known as Our World as Math. Every day and in everything we do we are exposed to the world of math. Shapes of every kind help guide our tasks. Can you imagine our world without numerals or a system without measurements? How would we know how much medication to give someone? Could we say, “Administer to him or her a little medication”? We need everyone to understand the same system of measurement.

Within this unit teachers and students will discover math concepts as students are paired with classmates. Activity Cards are available for students to choose. The unit is named Lime and contains activities regarding Shapes and Measurements. An Alphanumeric system can be found in the right hand corner of the card which helps the teacher organize the cards should they be printed out. Each card gives clear instructions on materials and the task. Students have available materials related to the activity card. These materials should be placed on a cart or table so students can readily access what is needed to complete the activity. The items on the cart might contain colored paper, scissors, colored pencils, glue, found materials, and journals. On another cart for this unit we find Montessori stencils, geoboards, Geofix sets, a Trundle Wheel, meter sticks, a French curve, and other items related to shapes and measurements the teacher wishes to include. There are available in the unit photo’s demonstrating what the Shapes and Measurement material center looks like. Some materials are clearly labeled, “choking hazard” even though students using this unit are between eight and ten years of age.

The math lab is designed to take place in a quiet environment with no “busy” work or worksheets available. Black line masters or “Nets” as they are known in the math lab are available to students and can be found in plastic bins. Each net relates to an activity card and when completed are displayed. The teacher joins students at some point to encourage journal writing or to present questions.

All of the math lab materials have been tested in a classroom setting over a period of three years. Students enjoy and remain engaged in the activities and improve math and writing skills. Question marks are used within the unit to encourage students to ask themselves questions, and stimulate creative thinking.

Lesson 1: Meet the Pattern Blocks!

Lesson Descriptors: Students recognize and form tessellations with pattern blocks.

Tessellation requires a covering of an infinite geometric plane without gaps or overlaps by congruent plane figures of one type or a few types.

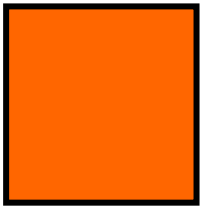
Focus: Identify the six pattern block shapes that tessellate.

Construction:

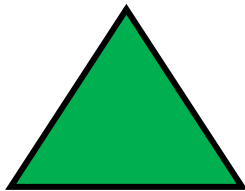
Materials—pattern blocks, colored pencils, manila drawing paper, and possibly a pattern block template.

Teacher

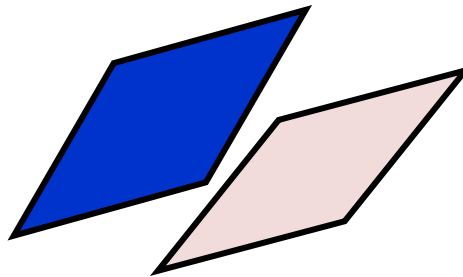
1. presents the pattern blocks one at a time and names them.
2. asks students to sort the entire container of pattern blocks by shape.
3. asks children to name the shapes.
4. offers drawing paper, colored pencils, and encourages that children trace a design to fill the paper.
5. suggests to students to explain their work.



square



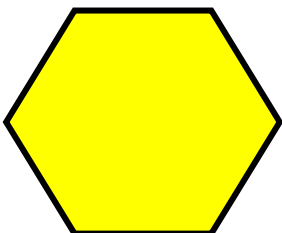
equilateral triangle



narrow tan rhombus or diamond
wide blue rhombus or diamond



trapezoid



hexagon

The six pattern block shapes have faces that are polygons or plane (flat) shapes with straight sides.

Assessment: The children's ability to name the pattern blocks, sort them, and design with them represents the familiarity that we seek.



Connections: Children are exploring shape and space in their compositions. This experience will evolve into tessellations, a contemporary art form.

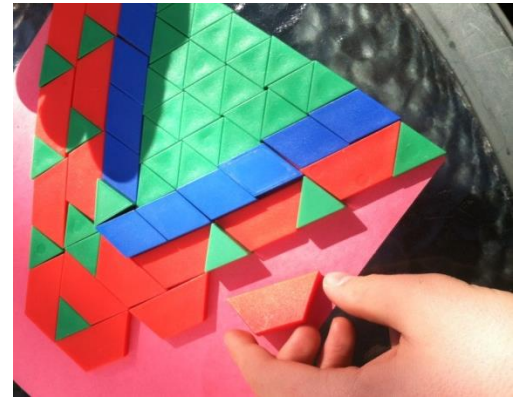
Lesson 2: Tiling or Tessellating with Pattern Blocks

Lesson Descriptors: spatial relationships, differences in shapes

Focus: Create patterns by tessellating, covering area with no gaps or overlaps.

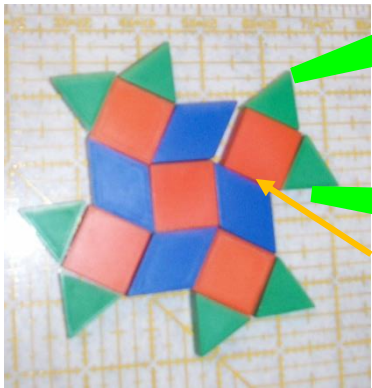
Construction:

Materials—pattern blocks, one-inch paper grid, colored pencils



Teacher

1. arrays a few pattern blocks, asking students to name them for review.
2. explains that for centuries people have used tiles to cover walls and floors.
3. challenges students to cover the grid with designs.
4. Teacher asks students to choose the favorite part of the design to trace and name the shapes.

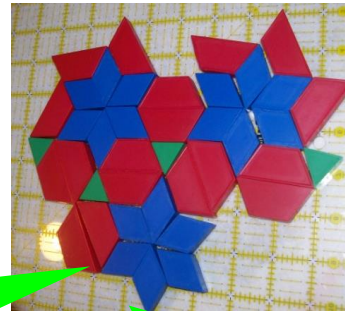


green equilateral triangle

orange square

red trapezoid

blue rhombus or parallelogram



The pattern blocks tessellate or tile—they fit together.

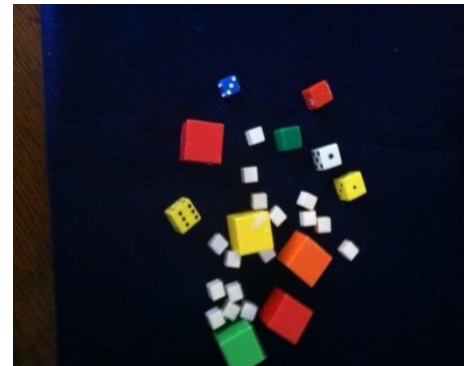
Assessment: As students share their work they will show their vocabulary and spatial concepts.

Connections: Students should be encouraged to view the work of M.C. Escher online to see how far an artist can take tessellations. They may want to see some ancient tile work as well.

Lesson Plan3: Construct a Cube from a Net

Lesson descriptors: solid shape vocabulary, understanding the properties of the cube, development of eye-hand coordination, decorating the cube as a creative outlet

Focus: Children relish making solids from nets. This process brings along with it vocabulary and concepts.



A solid shape has three dimensions of length, width, and depth!^T

A polyhedron is a solid shape with faces that are all polygons.

A net is a plane shape that can be folded into a solid shape.

A cube is a polyhedron.

Construction:

Materials—black-line master, a net for the cube, a set of solid shapes, a set of polygons; crayons, scissors, glue sticks, and binder clips.

The teacher

- 1. presents a set of solid shapes and ask how they are different from the polygons and present the polygons for comparison; define solid shape and polyhedron as above.**
- 2. shows the black line master, the net; asks how it can be turned into a solid shape, and what solid shape is it likely to be.**
- 3. shows the students how to cut, glue, and clip the cube together (the cube will benefit from being clipped overnight)**

Assessment:

The teacher can assess the students' dialogue for the inclusion of new vocabulary; assess the logic of how they construct the project; and finally the outcome of the project.

Connections:

Seeing the distinction between plane and solid shapes is important in all forms of construction as is the particular solid called the cube.

Lesson Plan 4: Make a Snowflake & Measure Angles

Lesson Descriptors: studying in historical context, seeing the hexagonal structure of snowflakes, seeing the combined effects of folding and cutting, introducing angle measurement, developing eye-hand coordination

Focus: discover the importance of the consistent structure of snowflakes.

Construction:

Materials— paper lace doilies, other colored or metallic paper, scissors, protractors



The teacher

1. shows examples of snowflakes from pictures or real life; ask students to make observations of similarities and differences.
2. shares the history of snowflake research and as noted below
3. gives the snowflake directions and explanatory material to the students in reproduced form
4. collaborates with the students as they make their snowflakes and demonstrates how to use the protractor



All snowflakes that we know of have six points like a hexagon, but, otherwise, they are all different.

Wilson A. Bentley spent much of his life (1865 to 1931) photographing snowflakes with a camera and microscope.

You can see snowflakes with a hand lens—better on a dark background!

1. With a teacher, fold a round paper lace doily in half.
2. Fold one folded side from left to right so that three 60° angles appear.
3. Fold again from right to left so that you have the three 60° angles layered.
4. Fold in half again forming six 30° angles, layered.
5. Cut away much of the folded edges; open out for a hexagonal snowflake!
6. Repeat with friends for a winter display? Glue onto colored background?

T-Note: Here is an *experience* with angle measurement in craft. Also try a square of paper folded in the same way and then cut off in an arc around the outer edge. Try different papers and colors. Glue onto card stock for greetings? See Ken Libbrecht's work online.

Assessment: Students' dialogue will reveal enthusiasm for and understanding of the history of snowflake research. The teacher can take note of any needs for additional practice with scissors and paper and angle measurement with the protractor.

Connections: Students interested in snowflakes and Bentley's work should be encouraged to research further in the internet. This is both historical and scientific research. Students can look up Bentley's home in Vermont. Making snowflakes of different materials should be encouraged.

Lesson 5: Construct a Tangram Puzzle

Lesson descriptors: polygon vocabulary, creative rearrangement of elements, spatial awareness, sensing self-correcting closure

Focus: allowing the student to create something new out of known shapes

Construction:

Materials—black-line master for tangrams, scissors, glue sticks, colored paper for background

The teacher

1. shows students examples of completed tangrams.
2. shares some tangram history as suggested below.
3. explains that tangrams have seven tans that make it a complete puzzle.
4. distributes black-line masters and invites students to cut them apart and reassemble them in several ways.



The tangram puzzle has seven pieces called “tans.” There are five triangles, a square, and a parallelogram. Historians believe that the puzzle was designed long ago in China, but they are not positive. Students may be interested in researching online. The pronunciation is: accent on the first syllable and a very short “e” *schwā* sound in the second syllable. There are many sources of tangram designs, so check at the library and online along with School Specialty as below.

www.didax.com 2-490J [4 tangram puzzles] www.schoolspecialty.com 1016623 [*Tangrams from a Jar*, design cards]

Assessment: Teacher can observe the flexibility of thinking in the variety of designs, use of all the pieces, and general awareness of space. Teacher can listen for acquisition of tangram vocabulary.

Connections: The research suggested above is clearly social studies; the creative problem solving is scientific.

Lesson Plan 6: Create a Storybook Character from Tangram Pieces

Lesson descriptor: seeing possibilities in shapes, turning to resources for inspiration, seeing the camera as a tool for data collection

Focus: to see basic shapes imaginatively

Construction:

Materials—purchased plastic tangrams, as itemized below, a library of storybooks, a teacher with a camera, black-line master of assembled tangram shapes

Teacher

1. reviews the origin of the tangrams, name the shapes, show some animal character designs.
2. distributes black-line masters and sets of plastic tangrams.
3. invites the students to recall some favorite animal storybook characters or research some in the classroom library or online.
4. invites students to array the tans to resemble the characters.
5. explains that the teacher is ready to record designs of student work and help record a story to go with design.



A tangram puzzle has seven pieces called “tans”--five triangles, a square, and a rhombus. Many purchased plastic tans come in three primary colors and green. We keep colored felt cut to one square meter always available. Felt and paper make the manipulatives quieter and more visible. This activity can be recorded by tracing the tans. Some students find this tedious, so we suggest the camera.

www.didax.com 2-490J [tangrams], www.schoolspecialty.com [1016623 *Tangrams from a Jar*, design cards]

Assessment: Teacher can observe the interest and excitement of students when addressing a very open-ended question, the breadth of inquiry in finding stories to illustrate.

Connections: This lesson can stimulate the students to repeat the experience and build a portfolio. It may also stimulate interest in researching forms of book illustrations.

Lesson Plan 7: The City House

Lesson descriptors: Seeing possibilities in shapes, investigating resources for information regarding shapes in structures, and creating individual design.

Focus: Discovering shapes in the real community



Robert Frost Home

Construction:

Materials—colored paper, scissors, crayons, glue, and a variety of pictures of buildings for additional design

Teacher

1. discusses with students urban communities and building designs, contrast and compare urban and rural communities.
2. invites students to describe the types of homes in their communities. This includes the age, size, shapes seen, type of properties the house and surrounds have.
3. distributes the black-line master The City House.
4. invites students to design the city house.
5. invites students to discuss the different shapes they see as they constructed the house.

Assessment: Teacher listens as students identify shapes identified within the city house project. Teacher tape-records students describing homes and buildings in their community.

Connection: This activity can stimulate students to investigate further urban life, famous buildings in America, investigate historical dwellings in their community/cities.

Students can investigate the works of Robert Frost, Frank Lloyd Wright, or the history of the Empire State building, when it was constructed, and who owns it.

Lesson Plan 8: Polygons with Messages

Lesson descriptors: Identify polygons by the number of sides they have. There are 10 sided, 4 sided, 6 sided that students name

Focus: Children make connections between shapes and what they say. It is fun to create their own.

Construction:

Materials—colored pencils, scissors, glue, and construction paper

The teacher

1. reviews the definition of a polygon. Question students to ask where they have seen these shapes used?
2. asks students to cut out triangle, pentagon, hexagon and use the colors of the signs that are found in communities
3. encourages students to create their own signs with the use of the hexagon, quadrilateral, and pentagon. What message would they like to display on their shape?
4. suggests students design a mural and share their created signs. Encourage students to construct a story to place on the mural.

Assessments: The teacher listens as students explain what shapes they used in the mural and what they named the hexagon, quadrilateral, and pentagon.

Connections: seeing the connection between the shapes and the messages they give.



Have students write a description of a quadrilateral without naming the shape. Remind them to include enough details so that anyone reading their description can draw the quadrilateral.

Measures of Volume: Double Hummus^T Lime 3.7.3Kite



Ingredients for about four servings:

400 ml of cooked chickpeas^T

40 ml of plain yogurt^T

25ml of lemon juice^T

25ml of oil^T

curry powder^T

garlic or garlic powder^T

salt and pepper

Doubled:

* 2 = 800 ml

* 2 =

* 2 =

* 2 =

Garbanzo (gar-bahn-zo) is the Spanish word for the same beans that are called ceci (chay-chee) in Italian and chickpeas in English.

With a teacher, decide to make one recipe of hummus or to double it. To double, fold a journal page in half vertically, copy the recipe as above, and complete the doubling.

Rinse the beans in a colander; transfer to a wooden bowl; mash with a potato masher.

Add the other ingredients; mix and mash some more.

Scoop with crackers, chips, or crusty bread; eat!



T-Note: This activity demonstrates the ease of changing recipes in base-ten metrics. Any white beans can be substituted. With concerns about canned foods and hazards of opening cans with students, teachers may decide to soak and cook dried beans ahead. The yogurt replaces the traditional tahini (sesame paste)--more costly, less nutritious. The lemon juice can be squeezed ahead of time by students? Olive oil is most flavorful but others work--We try for non-genetically modified. Fresh raw garlic is known to enhance the immune system. We have successfully peeled and minced it with plastic knives that tend not to cut fingers. Any combination of tropical spices like curry or chili power will add flavor.

Measures of Volume: Making Granola^T

Lime 3.7.3Kitc

T-Note: We offer cooking as learning activity in many series for early school-age children. The teacher's judgment about student maturity and safety determines whether to do the cooking while students watch or with students as an activity in measuring volume, temperature, and time. Teachers are cautioned also to check student records and organizational policies for allergies especially to nuts.

Equipment:

a rectangular baking pan
a 1-cup measure^T
an oven
a spring-loaded timer^T
an oven thermometer^T

oven thermometer



spring-loaded timer



Ingredients & directions:

Preheat the oven to 300° F.
Pour $\frac{1}{2}$ cup of vegetable oil into the pan.
Heat the pan in the oven, time for 10 minutes.
Add 2 cups of oats, 1 cup of wheat flakes,
1 cup of rye flakes.
Stir, bake, time for 15 minutes.
Add your favorite nuts or seeds or both.
Stir, bake, and time for 10 minutes.



T-Note: This is measuring capacity, time, and heat--USC and Fahrenheit measurement as they are found in cookbooks across the US. However, we always recommend equipment that has USC and SI, Fahrenheit and Celsius gradients because students will notice the benchmark comparisons. The spring-loaded timer allows students to sense the effect of winding it up and letting it unwind, clicking away the time; the oven thermometer is a separate item that students can examine before it goes into the oven—both from local stores. Ingredients are available from health food stores. The recipe is adapted from *Joy of Cooking*—See *Bibliographies*.

Measures of Volume: Make Soft Clay?^T Lime 3.7.2 Kitb

T-Note: We include recipes for kitchen clays in all series for young children. Because this activity involves **cooking**, the teacher's judgment about **student maturity** and **safety** determines whether to make the clay ahead and offer it to children as art material, make it while students watch, or make it with students as a **shared activity** involving math.



Two friends will need: a saucepan, a small mixing bowl and spoon, measuring cups and spoons.

In the pan, stir the dry ingredients:

1 cup of white flour
 $\frac{1}{2}$ cup of kosher^T salt
 $\frac{1}{2}$ tablespoon of cream of
tartar^T

In the bowl, mix the wet ingredients:

1 cup water and a little more?
2 tablespoons vegetable oil^T
a few drops of food coloring

Add the wet to the dry in the pan.

Cook^T and stir over medium low
heat until the dough balls
up.^T

Cool and begin modeling!!^T

For an eruption, mix 1 tablespoon of active dry yeast to 1 cup of hydrogen peroxide. Stir. Watch and wait a few seconds.

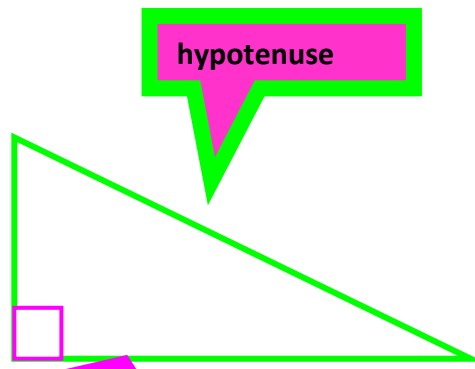
T-Note: We found that the last step takes about ten minutes and lots of muscle! With students look up the origins of “kosher” and “cream of tartar”? We make a point of buying veggie oil that does not come from bioengineered seed!! The clay lasts several days, then, must be discarded—It molds! Students can use the clay anytime for representing their work or as a creative arts material—works with plastic cookie cutters—Check for choking hazards for under threes!

NCTM-M-2

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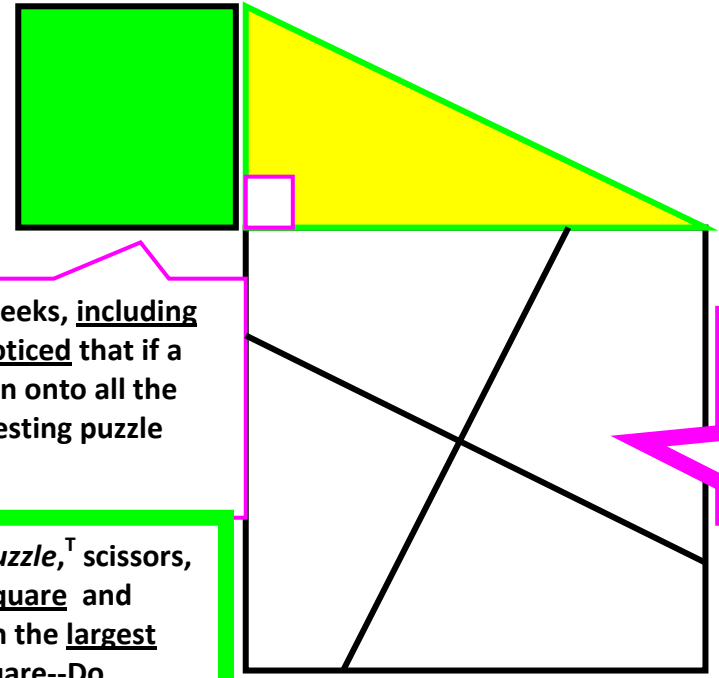
Measures: Right Triangles & Their Sides

Lime 3.2.2SM



A right triangle has one 90 ° angle. It has two sides called legs and a hypotenuse (high-pot-en-oos).

The ancient Greeks, including Pythagorus, noticed that if a square is drawn onto all the sides, an interesting puzzle appears.



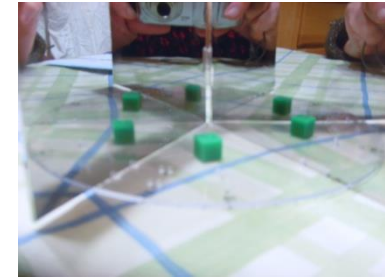
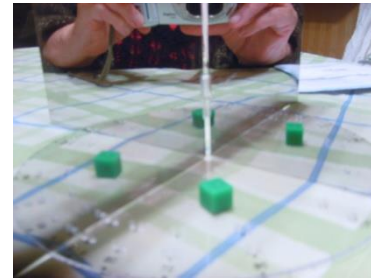
These two line segments are parallel with the sides of the upper square and the hypotenuse.

Two friends will need: copies of the Right Triangle Puzzle,^T scissors, and glue stick. Cut around the medium-size square and along its inner line segments; lay the pieces on the largest square. Can you find area for the smallest square--Do the broken line segments give a clue? With a teacher, glue the pieces in place on the largest square. Repeat in another color? Two colors? What conclusions can you draw?^T

T-Note: This puzzle is a proof of the Pythagorean Theorem: the square of the hypotenuse equals the sum of the squares of the legs. When letters are assigned to the three sides, a formula can be written as $a^2 + b^2 = c^2$ or $c = \sqrt{a^2 + b^2}$, typical of high school algebra. However, all observations are welcome here and can be recorded by the teacher. SM 15.3.6Dr NCTM-- ©2013

Measures of Angles: The Angle Mirror

Lime 3.2.2AM



Two friends will need: an angle mirror^T and protractor, pattern blocks, and a teacher with a camera.

Younger student, place an orange square into the angle vertex with both mirrors at 45° . Four squares? One big square? Pull the block away from the vertex? Changes?

Older student, repeat with a green triangle or any small block? How many shapes do you see with both mirrors at 45° ? 60° ?

Both students, with a teacher, record your findings so far. Repeat, taking turns, with all the block shapes. Reflect them with the mirrors at 45° and 60° ? Similarities? Differences? Move only one mirror?

Record with a teacher and camera?

^T-Note: We make angle mirrors by taping together two unbreakable mirrors with colored masking tape; we match them to protractors of similar breadth. The protractor too can be made of card stock with markings for 30, 45, 60, 80, and 90 degrees. The tighter the angle, the more reflections. Students delight in observing the principle of the kaleidoscope!

^Swww.didax.com 2-504J [pattern block mirrors], 197893J [6-inch protractors]

NCTM M 2

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Measures of Length: Perimeter & Estimation Lime3.2.1TW



Perimeter comes from old Greek *peri-* for “around” and *metron*^T for measure.”

Two friends will need: the trundle wheel, cm grid, and a teacher with an adding machine.

Both students, with a teacher,^T choose a large indoor or outdoor rectangular space; draw on grid.^T

Younger student, measure two sides with the trundle wheel;^T record on the drawing.^T

Older student, repeat for the other two sides.

**Both students, add the side lengths for the perimeter;
record on the drawing.**

Repeat, for another space.^T But this time, estimate, or guess, first. Write about what you did.



^T-Note: The drawing need not be to scale--taken up elsewhere in the math lab. We round these measures to the nearest meter using the term, “about” or “approximately?” We recommend adding on the machine to avoid distraction from the measuring. ⁵www.didax.com

Measures of Length: God Designs an Ark.

Lime 3.2.1FT

Genesis 6:15 tells us:

“The length of the ark shall be three hundred cubits,
the breadth of it fifty cubits,
and its height of it thirty cubits.”^T

There were to be three decks and rooms
inside, a window, and a door.

[Click to view video of this activity.](#)

A cubit is the length of the forearm from the elbow to the fingertips—about 46 cm.

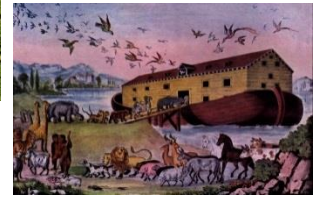
Two friends will need: twine,^T cardboard of 46 by 15 cm,
a large outdoor space, and drawing paper.

Younger student, wind 50 cubits of twine around the
cardboard measure .

Take turns, zigzagging the string to measure out for 300 cubits; form a cross of the two lengths of string.

Standing inside the imaginary ark, consider height of 30 cubits
and where people and animals were on three decks.

Draw and write. Where did Johan Huibers build an ark?



Measures: Metrics, US Customary & Length Lime 3.2.1FMd

Metrics are used around the world.^T
They are also called “SI” for *Système International* (sees-tem een-tayr-nas-ee-oo-nal)

≈ means “about equal to.”
> means “is greater than.”
< means “is less than.”

US Customary measures came from England but are used in the USA. They are also called “USC.”

A cm ≈ the width of your little finger.
A meter stick is 100 cm.

An inch ≈ the width of a man’s thumb.
A foot ≈ the length of his foot or 12 inches.
A yard ≈ the length of his outstretched arm, fingers to nose.

Benchmark comparisons:

A meter is 100 cm > a yard. A yard is 36 inches < a meter.
A yardstick is 36 inches often with 91 cm written alongside.
A metric ruler is usually about 30 cm long.

OBJECT	SI CM		USC IN
book	25 cm	≈	10 in

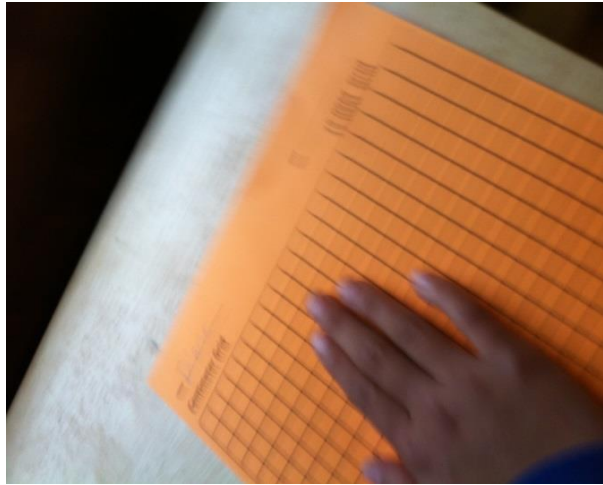
Materials: a metric ruler and meter stick, a USC ruler and yardstick, and long tape measure (USC-metric).^T

Array some long objects to measure for length.

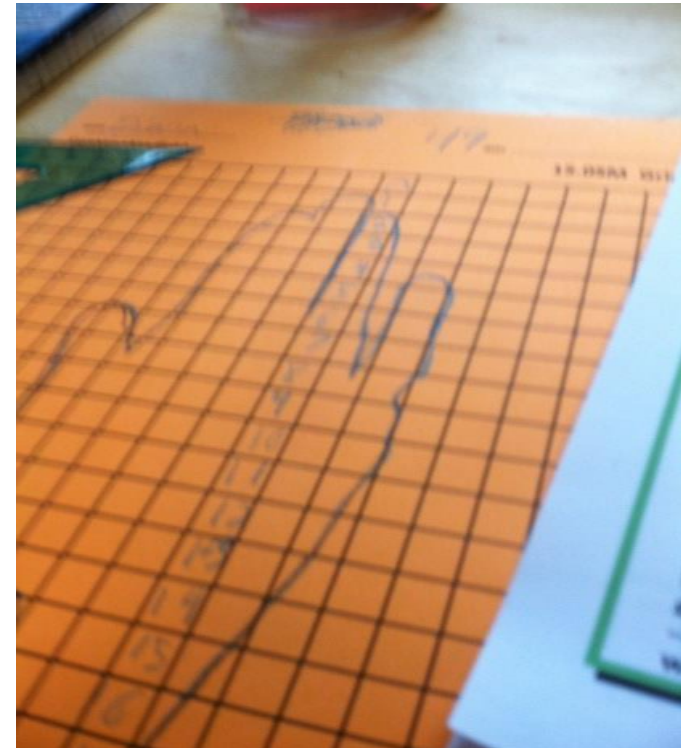
With a teacher, rule the table on a journal page. Measure, record with the symbols as above, and compare. Write about what you learned.

T-Note: Benchmarks (NCTM’s term) are points of comparison, between metrics and USC--internalized painlessly in this and similar activities. Some students will notice the base-ten structure within SI: Each measure is ten times the next smaller one and one tenth the next larger one. So, a centimeter (cm) is ten times a millimeter (mm) and one tenth of a decimeter (dm). That is also one tenth of a meter (m). Others will learn with *shared experience and dialogue*. SI in French is International System of Units in English. For safety rulers, see School Specialty.com NCTM M 2 ©2013 MLA

Measures of Length: How Big are Your Hands?



Time 3.2.1FMc



Two friends will need: cm grid.

Older student, with grid horizontal, put your name at right, your friend's left.

Younger student, put your right hand on the left side of the grid, fingers together.

Older student, trace the hand. Then, put your right hand on the right side for your friend to trace.

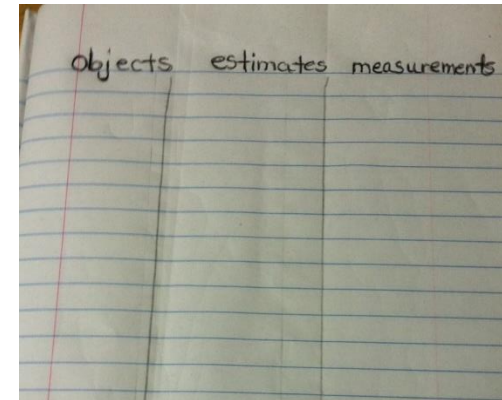
Both students, with a teacher, discuss--How big are your hands? Discuss length and width.^T

How do we define the hand--Where does it begin and end?^T How precise is our measure? How does the grid help?^T Write; display?

T-Note: This activity takes student interest in their growing bodies to the issue of precision in measurement. The need for an agreed-upon and precise starting point will arise often in measuring activities, so discussion is appropriate here. If students seem to be thinking intuitively about area, the teacher can name the concept as "the space within a flat shape" and ask how do the squares on the grid help to think about area?

Measures of Length: Circumference of a Basketball^T

Line 3.2.1FMb



Two friends will need: a basketball, meter stick or yardstick, about two meters of string, ribbon, or yarn, and a bit of masking tape.^T

Both students, fold a journal page into three columns; label the columns for objects, estimates, and measurements. Enter basketball as your first object.

Younger student, estimate the ball's circumference; enter your estimate into the column with the unit of measurement —cm or inches. Tape the string (ribbon or yarn) onto the ball; draw it around the circumference of the ball—the widest possible measure? Remove, measure, and record with the unit of measurement.

Both students, with a teacher, discuss what else can be measured this way? Make four or more entries in your columns?^T

**Circumference is the distance around a circle or its perimeter.
How can the string help in measuring?**

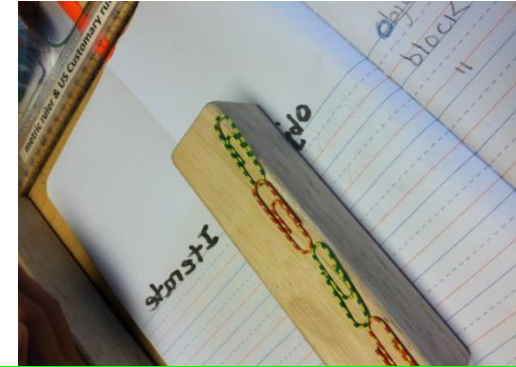
^T-Note: A measuring tape will work too! For other round objects, the teacher can suggest round waste baskets, lids to various containers, layer cake tins, the geometric solids, and so on. The teacher can suggest that a meter of string can be used as a measure anywhere. Students may enjoy researching history of the surveyor's chain!

NCTM M 2

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Measures of Length: Non-Standard Units

Lime 3.2.1FMa



We can iterate, or measure, with non-standard units—small but equal paper clips, toothpicks, and so on.

If you estimate, or guess, before iterating, your brain is working harder. Measure and compare with your estimate?

Record with units of measurement!

Two friends will need: objects to measure for length
and non-standard units—toothpicks?^T Clips?

Both students, fold the edges of two facing journal pages to the center for four columns; enter the headings for object, unit, estimate,^T and iteration.

Younger student, choose an object to iterate and a non-standard unit to iterate with; make these two entries into your journal columns. Enter an estimated measure; iterate and record.

Both students, take three turns each, write with a teacher.

T-Note: The purpose here is to demonstrate that all of our units of measurement were made up by humans throughout our history. Further, we have to resort to non-standard measures at times. For instance, to get a rough idea of the size of a piece of property, we may “pace it off” knowing that everyone’s pace is a little different! For students choosing toothpicks, the teacher should identify the hazard of sharp tips.

Measures of Length: Iterate Rooms and Hallways.

Lime3.2.1CCc



In a plane rectangle, either dimension can be called “width” or “length.”
These students are iterating the long dimension of a hallway as length.
They will iterate across the hallway to find the shorter dimension—width.

Two friends will need: connecting cubes,^S drawing paper of 9 by 12 inches^T, and a metric-USC ruler.^T
Younger student, with a teacher, choose a rectangular room or hallway; decide which dimensions of the room to call “length” and “width.”^T
Both students, iterate in connecting cubes from one corner of the room to the next—to the right or left?
To record: Reason which side of the rectangular paper best represents your iteration;^T write numerals with the unit of measurement (CCs for short?)
Repeat for the opposite dimension. Can you repeat for height?^T Illustrate with drawings of people?
Furniture? Write about what you did.

T-Note: Here is a chance to discuss dimensions of common papers—to date, all in USC! So we call for the metric-USC ruler. Teachers can explain the meaning of “by” in this context. Students may want to use the ruler and pencil to draw a rectangle that represents the room better than the sides of the paper. Some students may actually succeed in building a tower to the ceiling in one corner of the room for height. The CCs are 3/4 inch; some students can understand an explanation of the fraction.^S www.didax.com 2-300J NCTM M 1 ©2013 MLA

Measures: How Tall Are Your Friends?^T

Lime 3.2.1CCb



Two friends will need: a big tub of connecting cubes^S
and each other! **Perhaps** the **roll** of **mural** paper.^T
Younger student, **what's** the **best way** to **measure** your **friend**
in connecting cubes? **Ask** her or him to: **stand up**
while you make a **tall tower** to **match**? Lie **down** while
you **match** with a **row** of cubes? Or, lie on a big sheet of
mural paper while you **outline** the **body** in pencil and
then **iterate** the **length** in connecting cubes?
With a **teacher**, record your friend's name with the **iteration**
in connecting cubes of **height**? Same as **length** here?
Older student, repeat changing places. **Iterate** other friends?

T-Note: Here are self and body awareness and consideration of the other. Students will do this spontaneously. However, some students may be shy of this activity—The teacher can suggest another similar activity.^Swww.didax.com 2-3013

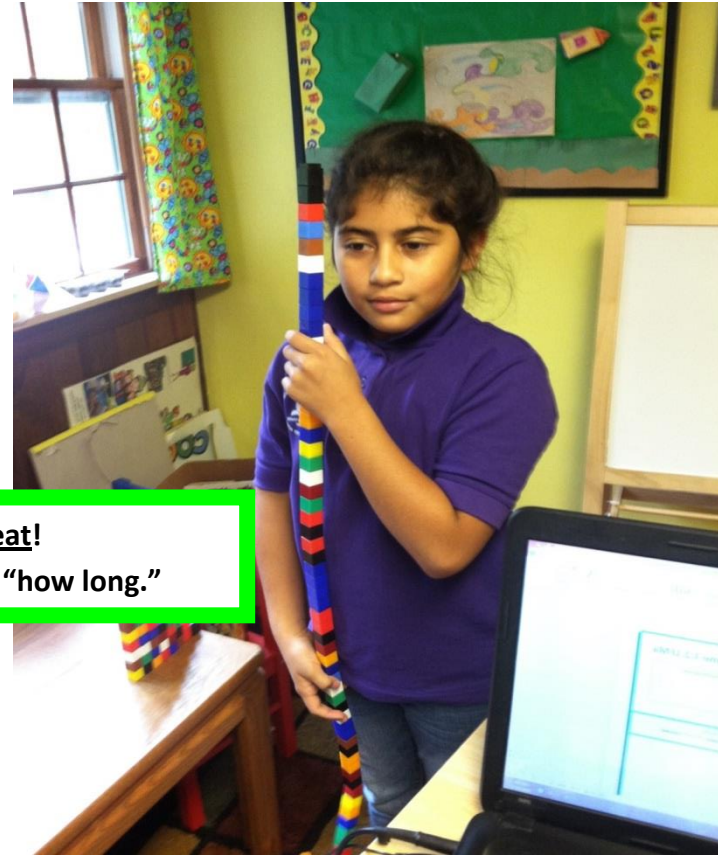
NCTM M 1

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Measures of Length: Iterate Objects.

Lime 3.2.1CCa



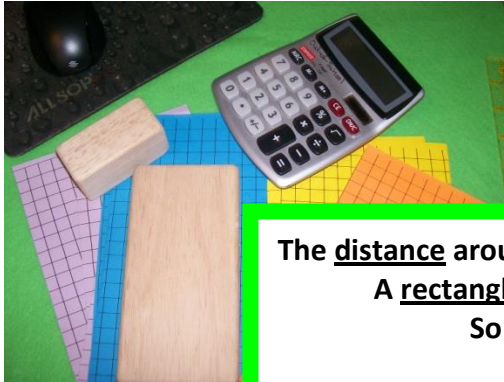
To iterate is to do something more than once—to repeat!
These students are iterating to measure “how tall” or “how long.”

Two friends will need: connecting cubes,^T some objects to measure like books, papers, and containers.^T
Both students, fold a journal page in half vertically for two columns; label one for object and the other iteration.
Record the name of an object and iterate for length; record with “CCs” as part of the entry? Exchange journals and check your data (information).
With a teacher, iterate the width of some of your objects, or height of chairs, or depth of a desk from front to back?^T

T-Note: The purpose is to define “iterate” almost the same as “measure” here. However, iteration suggests counting-on with many self-same units. Connecting cubes are 3/4 of an inch—difficult to explain to those unfamiliar with fractions. So, when students ask, we simply call connecting cubes, “non-standard units.” ⁵www.didax.com 2-300J[300 connecting cubes] NCTM M 2 ©2013 MLA

Measures: Perimeters of Quadrilaterals

Lime3.2.1FMb



The distance around a plane shape is called the perimeter.

A rectangle is a quadrangular plane shape with two pairs of straight parallel sides.

So its perimeter is the sum of the lengths of the four sides.

Squares are rectangles with four equal sides.

So their perimeters are four times the length of one side.

column addition:

6 cm

10 cm

6 cm

+ 10 cm

32 cm

Two friends will need: rectangular objects including something square,^T cm grid in colors, drawing paper, glue sticks, and a teacher with a calculator.^T

equation: $6\text{ cm} + 10\text{ cm} + 6\text{ cm} + 10\text{ cm} = 32\text{ cm}$

Each student, trace a rectangular object onto grid; cut it out and glue it onto drawing paper.^T Count the side lengths in cm and enter the measures with the unit of measurement (cm) along the sides.

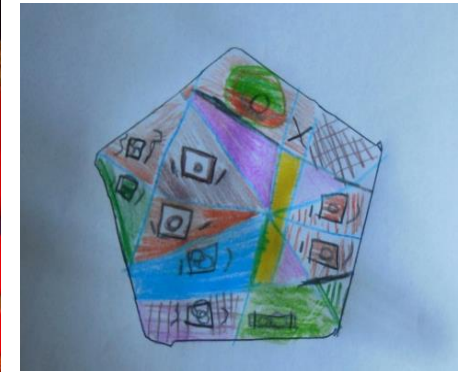
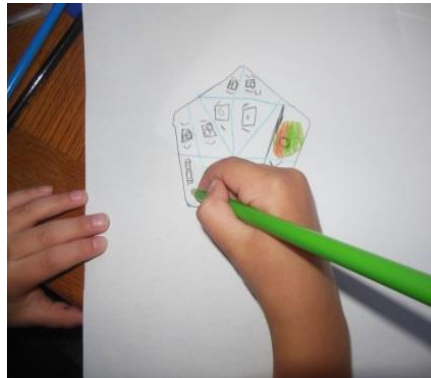
With a teacher,^T record the side lengths in a column and add for perimeter; check on the calculator and record as an equation.^T Check again, counting cm units all around? Repeat for a square object, a parallelogram that is not a rectangle (without square^T corners), for a rhomboid parallelogram (four equal sides without square corners). Display?

T-Note: We “round” to the nearest cm saying “about” or “approximately.” The calculator helps students unfamiliar with addition in two or more place values. If students appear ready to see the inside squares of grid as area, the teacher can explain that counting them means “area” in “square centimeters.” “Square corners” becomes “right angle” in other SACs. NCTM M 2 ©2013 MLA

Section 1

Measures: Volume, Angles, and Length

Plane Shapes: Montessori Stencils & Insets Lime 3.1.1TSb



Dr. Maria Montessori designed shapes for her students for matching and tracing.
The faces of these shapes are plane or flat.^T
Some are polygons with straight sides.
When you trace around them you are forming closed curves.

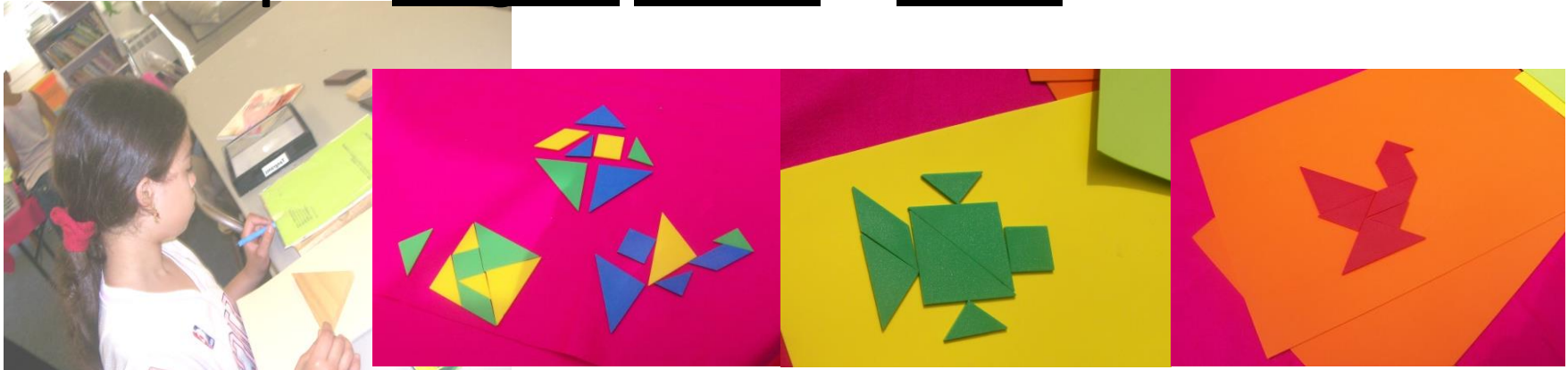
Two friends will need: the pink Montessori stencils
and the blue insets, colored pencils, and
drawing paper or special smaller paper?
Trace the stencils first? Trace the insets next? Do
the pink stencils help guide the pencil?
Decorate with hatch in colored pencils?
With a teacher, form a booklet and write?^T



T-Note: Real plane shapes have only two dimensions—length and width. Very flat shapes like these are often called “plane” but are really solid shapes because they have some thickness. Pictured are plastic versions of Montessori’s “metal insets.” We keep them in a labeled container in the writing-arts center along with other shapes for tracing. We keep drawing paper cut to the outer size of the stencils in the same place—maybe in pink and blue? Hatching is drawing parallel line segments—a Montessorian technique of focused exploration. Work done thusly is worth collecting into little booklets and/or displaying. ^Swww.didax.com 8-1586H [Montessori shapes], www.nienhuis.com 0.046.00 NCTM G 1, N & O 1 ©2013 MLA

Plane Shapes: Tangram Puzzles in Plastic

Lime 3.1.1TGb



A tangram (TAN-grum) puzzle has seven pieces called “tans” --five triangles, a square, and a parallelogram.

Many purchased plastic tans come in three primary colors of red, blue, and yellow as well as green, generally called secondary.

Two friends will need: tangram puzzles in four colors, two copies of *Tangram Puzzle: Filled In* from *Spectrum Masters*, drawing paper of two colors, black felt,^T a teacher with a camera and printer?^T

Each student, chose a tangram of one color, array the tans on the *Spectrum Master* to be sure you have all seven tans; form a design on paper or felt?

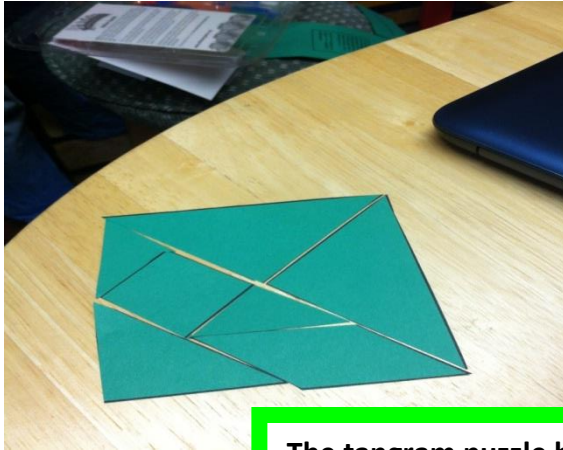
With a teacher, trace or photograph? Repeat in different tangram colors? Combine two colors but check on the *Master* for all the correct tans? Try different background colors? Print several photos on one sheet of copy paper? Write.



^TNote: We keep black and colored felt cut to one square meter always available. Felt and paper make the manipulatives quieter and more visible. Tracing is tedious for some students. .
^Swww.didax.com 2-490J [tangrams], www.schoolspecialty.com [1016623 [Tangrams from a Jar, design cards] SM 15.3.7Dra NCTM—G1

Plane Shapes: Tangram^T Puzzles in Paper

Lime 3.1.1TGa



The tangram puzzle has seven pieces called “tans.”

There are five triangles, a square, and a parallelogram.

Historians believe that the puzzle was designed long ago in China.^T

Each student will need: four copies of the *Tangram Puzzle*:

Filled In from *Spectrum Masters* (in red, blue, yellow, and green),^T scissors, glue stick, and large drawing paper.

With a teacher, fold the drawing paper in half horizontally and in half again vertically to make fourths—quarters or quadrants.^T Cut out a tangram square and its pieces.

With the drawing paper horizontal, array a tangram design in the upper left quadrant. Be sure to use all seven tans but not to overlap them. Leave some space below for a sentence? Glue in place. Repeat for the other three quadrants. Write for a display?

^T-Note: The pronunciation is: accent on the first syllable and a very short “e” schwa sound in the second syllable. The faces of the tans are polygons. Historians are not positive that the puzzle is Chinese in origin; their research, as published online, may interest students. The ideas of vertical, horizontal, fractions, and quadrants, may be new to some students, so we call in the teacher to discuss. There are many sources of tangram designs, so check at the library and online along with School Specialty, as below.

^Swww.didax.com 2-490J [4 tangram puzzles] www.schoolspecialty.com 1016623 [*Tangrams from a Jar*, design cards]

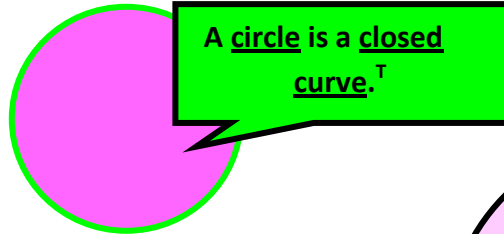
SM 15.3.7Dra

NCTM G 1

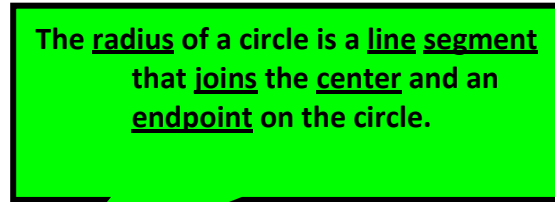
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Plane Shapes: Parts of Circles

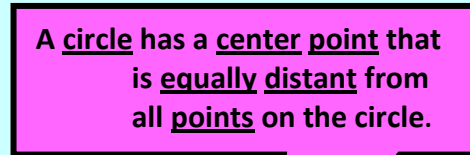
Time 3.1.1SCc



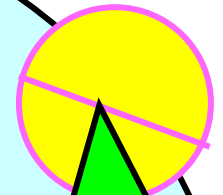
A circle is a closed curve.



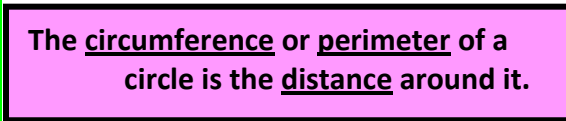
The radius of a circle is a line segment that joins the center and an endpoint on the circle.



A circle has a center point that is equally distant from all points on the circle.

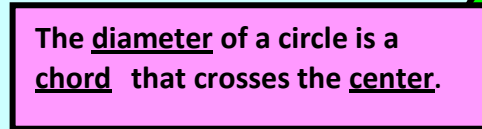


A chord is a line segment that joins two endpoints on a circle.



The circumference or perimeter of a circle is the distance around it.

Two friends will need: safety compasses,^T drawing paper, colored copy paper, metallic paper and glitter?^T
Draw some circles of all sizes, some overlapping?
With a teacher, label centers, circumferences, chords, diameters, and radii.^T



The diameter of a circle is a chord that crosses the center.

^T-Note: The purpose here is to familiarize students with circle vocabulary. The plural of radius is radiuses or radii. We prefer radii and pronounce it ray-de-ee. We believe that students gain from learning the classical Latin. The scholarly definition of circle is: the set of all points in a plane that are the same distance from a fixed point called the center. NCTM—G4 ©2013 MLA

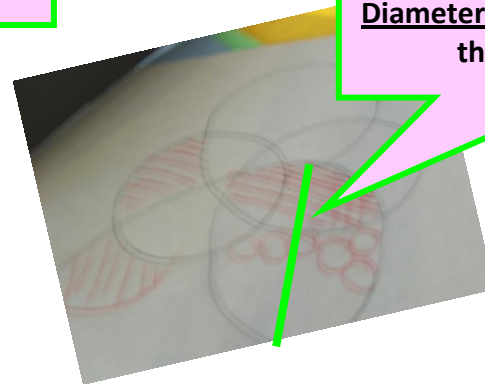
Plane Shapes: Draw Circles & Name Parts.

Lime3.1.1SCb

Circumference is the distance around the outside of a circle.



Diameter is the distance across a circle that divides its area in half.



The radius is the distance from the center point of a circle to any point on the circumference.
The plural of radius is radii but radiuses is also correct.

Two friends will need: safety compasses,^{T S} sharp pencils,
drawing paper,^T colors (pencils, crayons, water color
paint?).

With a teacher, practice with the safety compass; make
large circles? Small ones? Medium size?

Make some circles overlap others? Make concentric circles
—circles within circles.

Label circumferences, radii, and diameters.

Change papers with your friend? Fill with design?^T Color? Write?

T-Note: We recommend NO compasses with sharp points for elementary age students and know of none safe for children under three years; please see School Supply for safety compasses. The purpose is to learn vocabulary. Manila drawing paper (9 by 12 inches or larger) or copy paper will work. A few circular cut-outs of metallic or other interesting papers can lend interest.

^Swww.schoolspecialty.com

061350, 317835 [safety compasses for more than three years of age]

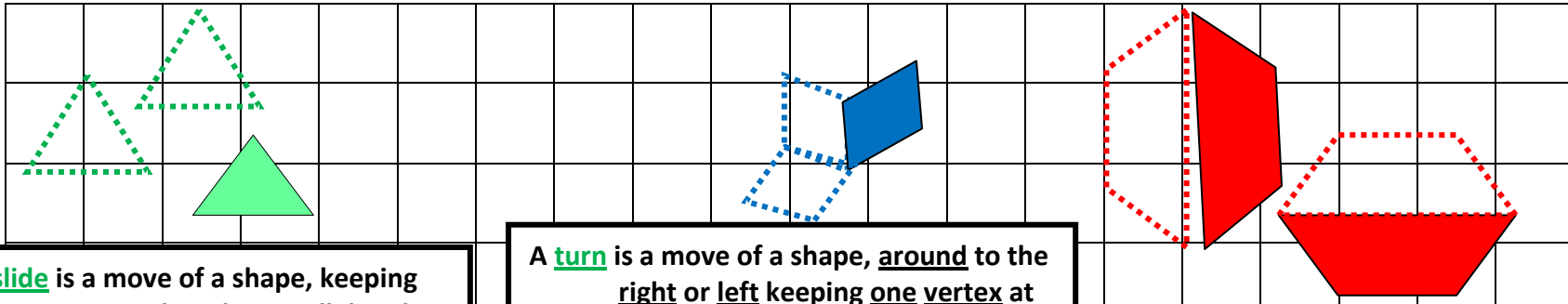
NCTM G 1, M 1

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Plane Shapes: Slides, Turns & Flips^T

Lime 3.1.1PBld

A rigid shape can be moved on a plane three ways called transformations.



A slide is a move of a shape, keeping its straight sides parallel with those in its starting position.

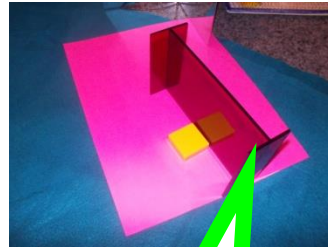
A turn is a move of a shape, around to the right or left keeping one vertex at the same point.

A flip is a move of a shape, picking it up and putting it down in a mirror reflection.

Two friends will each need: pattern blocks,^S matching colored pencils, inch grid,^T ruler, and geomirror.^S
Fold the grid in half vertically and horizontally—quadrants.
In the upper left, trace a green triangle, slide the block down and right so that it no longer overlaps; trace.
With a teacher, check for parallel sides; repeat in other directions?
In the upper right, trace a blue rhombus, turn as above, trace repeatedly to form a wheel?
At lower left, trace a red trapezoid, flip as above; check for reflection^T in the geomirror?
Write in the lower right quadrant? Repeat slides, flips, and turns with three other shapes on another grid?

^TNote: In math language, these transformations are called translations, rotations, and reflections.

Plane Shapes: Reflections in the Geomirror^T Lime 3.1.1PB1c



geomirror



Two friends will need pattern blocks, sheets of inch grid paper, and geomirrors.^T

each student, put a geomirror across a grid sheet, a block in front on your side. Is the mirror image the same as the block? Different?

Trace the block and its image!! Draw a line segment or a big "H" across the grid to represent the geomirror?

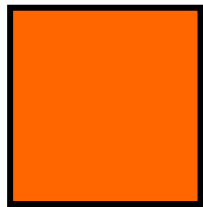
With a teacher, repeat for all the block shapes. How do they change in reflection?^T

Cut away excess paper; mount for display; write?^T

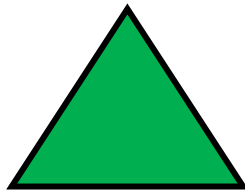
T-Note: Slides, turns, and flips can also be called translations, rotations, and reflections. A flip, the yellow hexagon looks the same but the green equilateral triangle may seem to "point" in opposite directions. We keep several geomirrors available in a labeled tote box. ⁵www.didax.com 210913J [pattern blocks], 2-516J [geomirror] NCTM G 2 ©2013 MLA

Plane Shapes: Meet the Pattern Blocks!^T

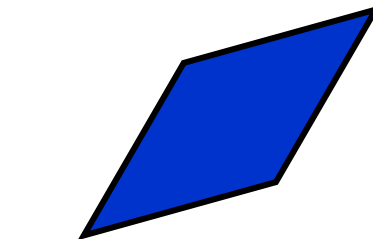
Lime 3.1.1PBl a



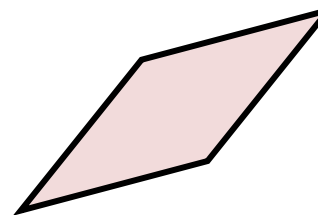
square



equilateral triangle



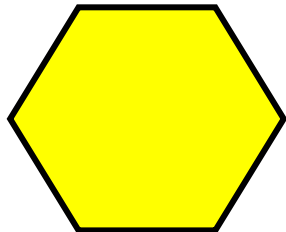
wide blue rhombus^T
or diamond



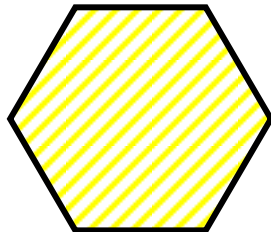
narrow tan rhombus or
diamond



trapezoid



hexagon



hexagon with hatching

The six pattern block shapes have faces^T that are polygons or plane (flat) shapes with straight sides.



Two friends will need: the pattern blocks,^{T S} colored pencils, journals, the pattern block template?^{T S}

Both students, trace and label one of each block—with the help of a template?

With a teacher, try some hatching; fill your paper with design; write.^T

^T-Note: The purpose is to explore the attributes (shapes and dimensions) of the pattern blocks. The pattern blocks tessellate—They fit together to fill two-dimensional (2D) space with no gaps and no overlaps. So we encourage students to fill out their designs. The block faces are plane (2D) shapes but the thickness makes the blocks solid shapes—3D. Most students can trace the blocks but, for some, the template helps by guiding the pencil. Templates are best stored in a wall file or teacher's tote. Webster's pronounces template with the schwa rather than the long a as in plate. The plural of rhombus is either rhombuses or rhombi. We prefer the latter classical Latin rhombi ending with the long “ee” sound. Hatching (a Montessorian practice) teaches focus.

Plane Shapes: Polygons, Eight Kinds^T

Lime 3.1.1GBb

Polygons are plane (flat) shapes with straight sides:

A trapezoid is a quadrilateral with two pairs of equal sides--one pair parallel.

A rhombus has four equal sides.

An isosceles triangle has three straight sides and two equal sides.

A parallelogram is a quadrilateral with two pairs of equal, opposite, parallel sides.

A hexagon has six equal sides.

An equilateral triangle has three equal sides.

A square is a rectangle with four equal sides and square corners.

A rectangle is a parallelogram with square corners.^T

Each student will need: square, circular, and isometric geoboards^S and dot matrices,^T colored elastics and matching pencils.^{T S}

Model a square on the square geoboard in colored elastic; draw on the square matrix in matching color; label as a square. Repeat for a larger square? Check for same number of dots and pegs, unit side lengths, and inside spaces in your drawings?

With a teacher and the same geoboards, repeat all for the other polygons with different colors — checking for part inside spaces too?^T Any that cannot be modeled?

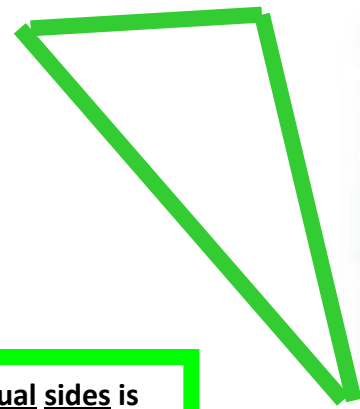
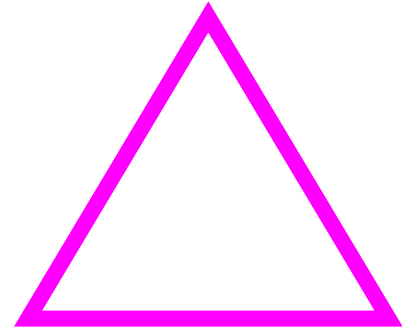
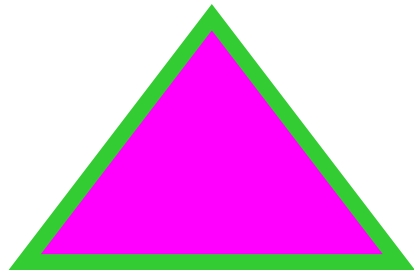
Repeat all with the isometric and circular geoboards.

Discuss, write, display?

^T-Note: The equilateral triangle, rhombus, and hexagon cannot be formed on a square geoboard.^S www.didax.com 2-420J, 210927J [geoboards] NCTM G 1 ©2013 MLA

Plane Shapes: Three Kinds of Triangles

Lime 3.1.1GBa



A triangle with two equal sides is an isosceles triangle.

A triangle with three equal sides is an equilateral triangle.

A triangle with three unequal sides is a scalene triangle.

Two friends will need: two square, 25-peg geoboards, two isometric geoboards, dot matrix paper to match both,^T six elastics of three different colors —colored pencils to match?

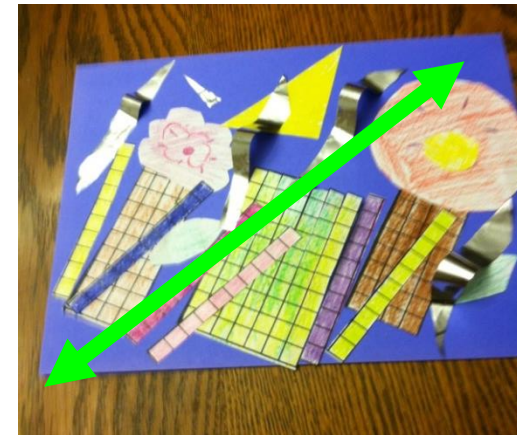
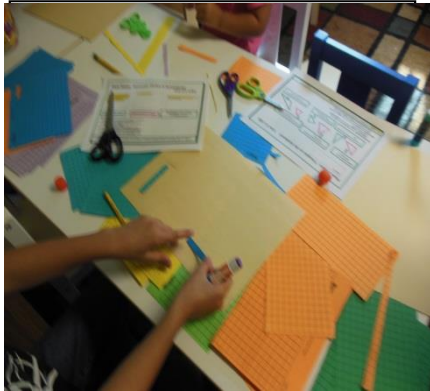
Both students, try forming the three types of triangles on the square geoboard. There will be one that cannot be formed. Which one?^T

With a teacher, draw on dot matrix in colored pencil; write. Repeat with a an isometric geoboard and isometric dot matrix?

^T-Note: The purpose is to see the possibilities and constraints of these matrices. The equilateral triangle cannot be formed on the square array geoboard because two sides would have to be longer or shorter than the pegs allow. The isometric geoboard will allow it. ⁵www.didax.com 2-420J, 2109271J [geoboards] SM 15.3.1DM, 15.3.4 NCTM G1 ©2013 MLA

Plane Shapes: Horizontal, Vertical & Perpendicular

Line 3.1.1C&G



Horizontal means across.

Vertical means up and down.

Perpendicular means crossing, with all corners the same size.

Two friends need: drawing paper, small pieces^T of cm grid of different colors, scissors, and glue sticks.

Cut strips of grid one cm wide and others two or three cm wide--all several cm long.

Array some onto drawing paper horizontally and some vertically—Are the strips now also perpendicular?

With a teacher, label and write for an artistic display? Add some strips of fancy paper? Glue everything in place.

A diagonal crosses your paper from corner to corner.

T-Note: The purpose is to establish meaning for these three terms common throughout geometry and measurement. Perpendicular can be defined as two line segments or shapes at right or 90° angles to one another. However, not all students will know how to check with a protractor. So, for this activity, we define perpendicular in terms of forming square corners. Students may enjoy adding metallic or textured strips of paper, cloth, or ribbon? We encourage filling in space in art work. We keep glue to a minimum—works best if weighted overnight! NCTM G--2 ©2013 MLA

Plane Shapes: Parallels, Intersects & Curves Line 3.1.1ASe

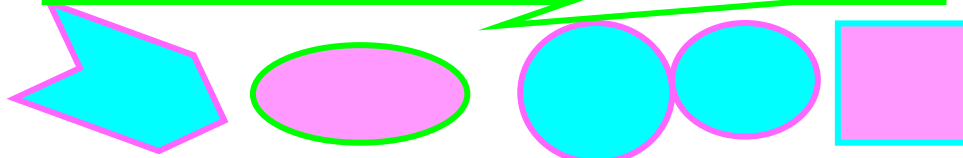
Straight lines have no angles (corners) or bends in them. We show them with arrowheads that mean they go on to infinity (∞). There are two kinds .

Parallel straight lines extend to ∞ (infinity) but never intersect (cross).^T

Intersecting straight lines do cross each other, but only once in all of ∞ .

A curve is any path (mark) on a plane (flat) surface that is drawn without lifting the pencil!^T It doesn't have to be straight! It can have bends and corners.

If the curve meets itself, it forms a closed curve.
Circles, squares, and figure eights are closed curves!!



Two friends will need: drawing paper and a straightedge.^T

With a straightedge, represent (draw):

two parallel lines;

two that intersect;

two more that curve.

Put arrowheads at each end to show that you're

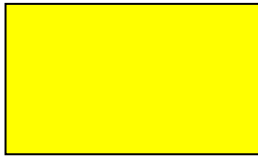
representing lines that go on to ∞ .

With a teacher, check the parallel lines with a ruler; label all.

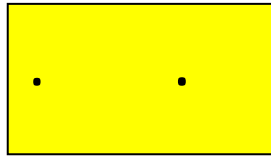
Draw some closed curves; fill in areas with color.

Plane Shapes: Points, Line Segments, & Polygons

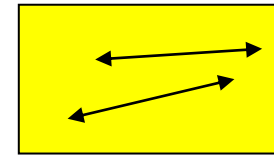
Line 3.1.1ASc



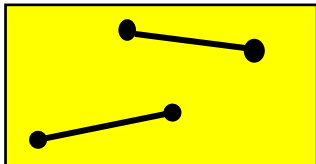
Points are invisible places in space.



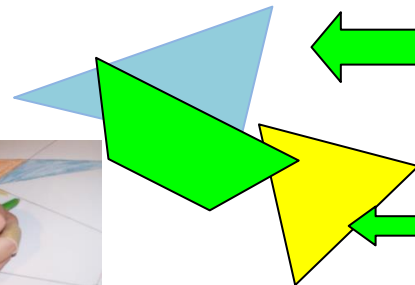
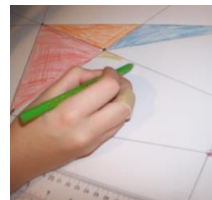
But we can represent them with pencil.



Lines are invisible rows of points that go on to infinity (∞) both ways. We show them with pencil.



Line segments are parts of invisible lines. The dots show their endpoints.



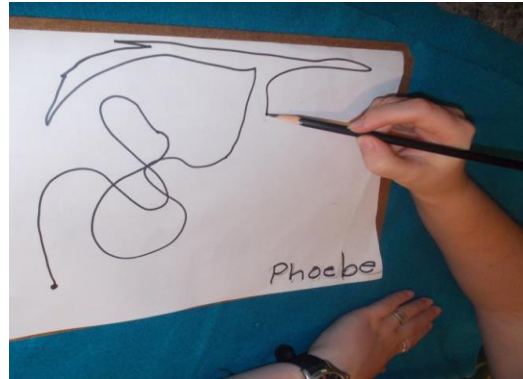
Three or more straight line segments connected form closed plane shapes called polygons.

This polygon is also a triangle.

Two friends will need: a straightedge,^T pencils, colored pencils, and drawing paper. Represent some points; connect for line segments;^T connect segments for polygons. With a teacher, extend segments to the paper's edges--imagine infinity!^T Color.^T

T-Note: Polygons too are really invisible because they have no thickness; as we add thickness with the pencil's graphite, we have constructed a three-dimensional (3D) or solid shape!! The straightedge can be any edge of a book or like object, but the ruler works best! Glitter glue suggests infinity! We insist that all or most of the drawing paper be filled with shapes and color. For safety metric rulers, see Didax.com or School Specialty.com.

Plane Shapes: Draw Curves & Closed Curves Lime 3.1.1ASb



A curve is a path that can be drawn in a plane, or flat, space without lifting a pencil.^T
A closed plane curve is a curve that begins and ends at the same point.^T
Artists combine curves and closed curves for interesting designs and

Two friends will need: two sheets of manila or white drawing paper,^T art pencils that leave a broad black mark, a clipboard, and fine markers?^T

Each student,^T put an art pencil at a point on drawing paper; move it along an interesting curve; turn so that you intersect with your own curve, forming a closed curve.

Without lifting pencil from paper, continue forming curves and closed curves to fill your paper.

With a teacher, look for trees, plants, buildings, people, or animals? Color the smallest closed curves and then some slightly larger ones until you're satisfied with your design?

^T-Note: These definitions are taken directly from Balka, Bana, Hoover, Marshall & Swan, *Visual Math Dictionary* (15,56), a wonderfully clear resource—See *Bibliographies*. Consider good quality paper for sharp definition of line and color. We keep felt markers apart from students except for special projects and offer only the fine, slender ones--Their vivid colors and inkiness can distract from the purpose of the work

Section 2

Plane Shapes

Plane Shapes: Tangram Puzzles in Plastic

Lime 3.1.1TGb



A tangram (TAN-grum) puzzle has seven pieces called “tans” --five triangles, a square, and a parallelogram.

Many purchased plastic tans come in three primary colors of red, blue, and yellow as well as green, generally called secondary.

Two friends will need: tangram puzzles in four colors,
two copies of *Tangram Puzzle: Filled In* from
Spectrum Masters, drawing paper of two colors,
black felt,^T a teacher with a camera and printer?^T

Each student, chose a tangram of one color, array the
tans on the *Spectrum Master* to be sure you have
all seven tans; form a design on paper or felt?

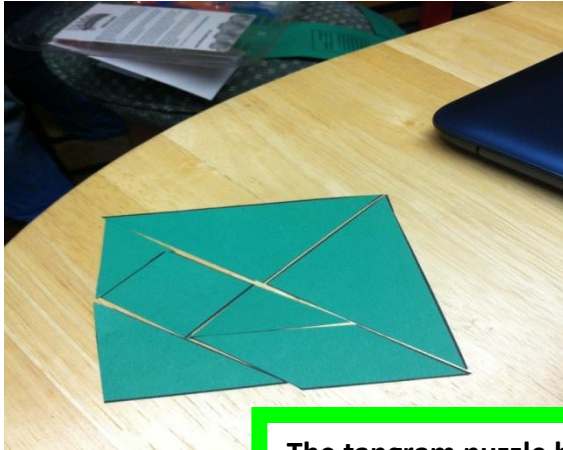
With a teacher, trace or photograph? Repeat in different
tangram colors? Combine two colors but check on
the *Master* for all the correct tans? Try different
background colors? Print several photos on one
sheet of copy paper? Write.



^T-Note: We keep black and colored felt cut to one square meter always available. Felt and paper make the manipulatives quieter and more visible. Tracing is tedious for some students. .
^Swww.didax.com 2-490J [tangrams], www.schoolspecialty.com [1016623 [Tangrams from a Jar, design cards] SM 15.3.7Dra NCTM—G1

Plane Shapes: Tangram^T Puzzles in Paper

Lime 3.1.1TGa



The tangram puzzle has seven pieces called “tans.”

There are five triangles, a square, and a parallelogram.

Historians believe that the puzzle was designed long ago in China.^T

Each student will need: four copies of the *Tangram Puzzle*:

Filled In from *Spectrum Masters* (in red, blue, yellow, and green),^T scissors, glue stick, and large drawing paper.

With a teacher, fold the drawing paper in half horizontally and in half again vertically to make fourths—quarters or quadrants.^T Cut out a tangram square and its pieces.

With the drawing paper horizontal, array a tangram design in the upper left quadrant. Be sure to use all seven tans but not to overlap them. Leave some space below for a sentence? Glue in place. Repeat for the other three quadrants. Write for a display?

^T-Note: The pronunciation is: accent on the first syllable and a very short “e” schwa sound in the second syllable. The faces of the tans are polygons. Historians are not positive that the puzzle is Chinese in origin; their research, as published online, may interest students. The ideas of vertical, horizontal, fractions, and quadrants, may be new to some students, so we call in the teacher to discuss. There are many sources of tangram designs, so check at the library and online along with School Specialty, as below.

^Swww.didax.com 2-490J [4 tangram puzzles] www.schoolspecialty.com 1016623 [*Tangrams from a Jar*, design cards]

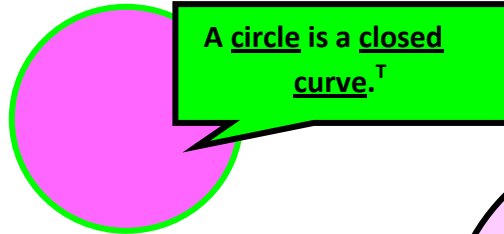
SM 15.3.7Dra

NCTM G 1

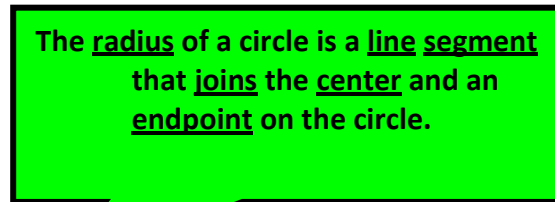
©2013 MLA

Plane Shapes: Parts of Circles

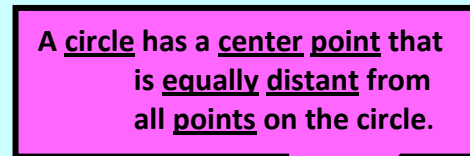
Time 3.1.1SCc



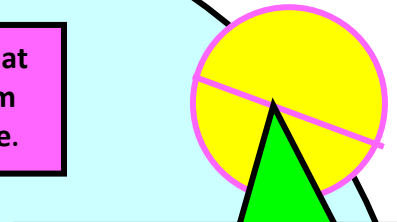
A circle is a closed curve.



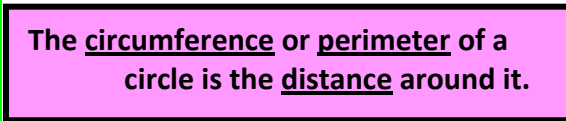
The radius of a circle is a line segment that joins the center and an endpoint on the circle.



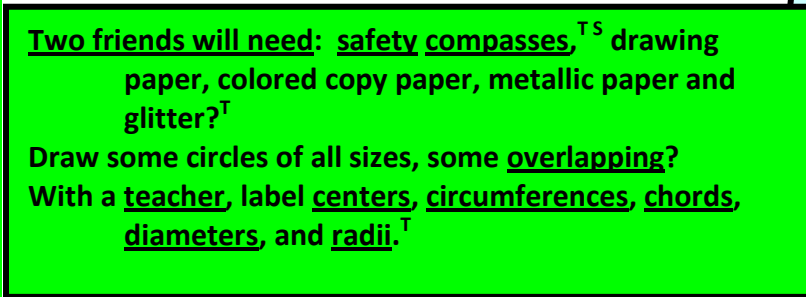
A circle has a center point that is equally distant from all points on the circle.



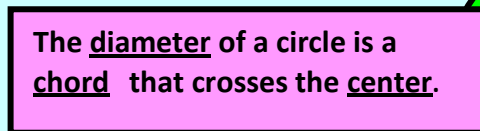
A chord is a line segment that joins two endpoints on a circle.



The circumference or perimeter of a circle is the distance around it.



Two friends will need: safety compasses, ^{T5} drawing paper, colored copy paper, metallic paper and glitter?
Draw some circles of all sizes, some overlapping?
With a teacher, label centers, circumferences, chords, diameters, and radii.^T



The diameter of a circle is a chord that crosses the center.

T-Note: The purpose here is to familiarize students with circle vocabulary. The plural of radius is radiuses or radii. We prefer radii and pronounce it ray-de-ee. We believe that students gain from learning the classical Latin. The scholarly definition of circle is: the set of all points in a plane that are the same distance from a fixed point called the center. NCTM—G4 ©2013 MLA

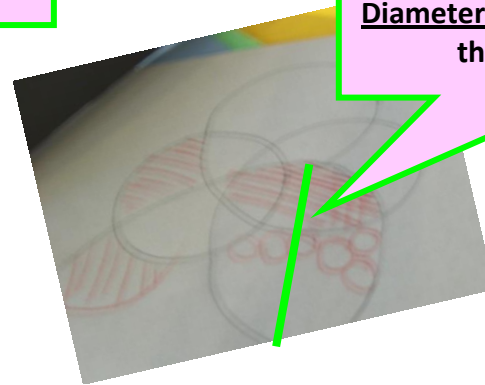
Plane Shapes: Draw Circles & Name Parts.

Lime3.1.1SCb

Circumference is the distance around the outside of a circle.



Diameter is the distance across a circle that divides its area in half.



The radius is the distance from the center point of a circle to any point on the circumference.
The plural of radius is radii but radiuses is also correct.

Two friends will need: safety compasses,^{T S} sharp pencils,
drawing paper,^T colors (pencils, crayons, water color
paint?).

With a teacher, practice with the safety compass; make
large circles? Small ones? Medium size?

Make some circles overlap others? Make concentric circles
—circles within circles.

Label circumferences, radii, and diameters.

Change papers with your friend? Fill with design?^T Color? Write?

T-Note: We recommend NO compasses with sharp points for elementary age students and know of none safe for children under three years; please see School Supply for safety compasses. The purpose is to learn vocabulary. Manila drawing paper (9 by 12 inches or larger) or copy paper will work. A few circular cut-outs of metallic or other interesting papers can lend interest.

^Swww.schoolspecialty.com

061350, 317835 [safety compasses for more than three years of age]

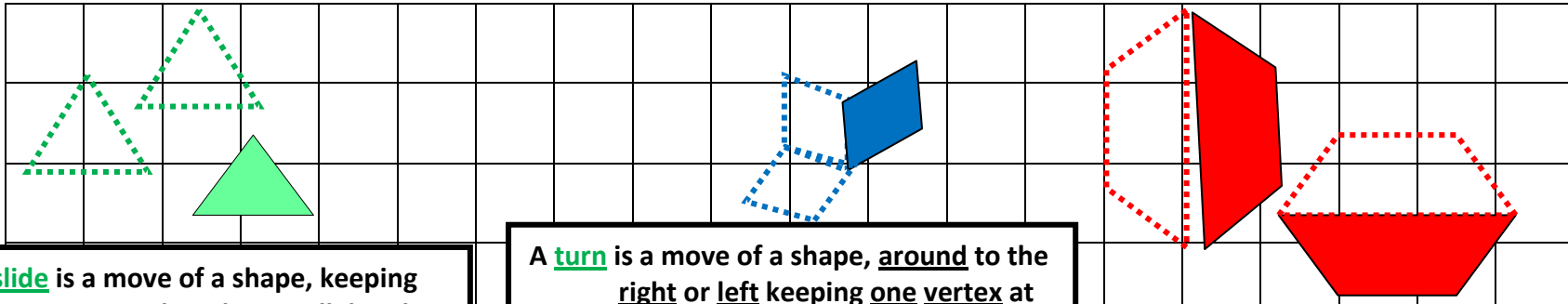
NCTM G 1, M 1

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Plane Shapes: Slides, Turns & Flips^T

Lime 3.1.1PBld

A rigid shape can be moved on a plane three ways called transformations.



A slide is a move of a shape, keeping its straight sides parallel with those in its starting position.

A turn is a move of a shape, around to the right or left keeping one vertex at the same point.

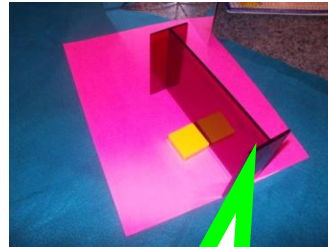
A flip is a move of a shape, picking it up and putting it down in a mirror reflection.

Two friends will each need: pattern blocks,^S matching colored pencils, inch grid,^T ruler, and geomirror.^S
Fold the grid in half vertically and horizontally—quadrants.
In the upper left, trace a green triangle, slide the block down and right so that it no longer overlaps; trace.
With a teacher, check for parallel sides; repeat in other directions?
In the upper right, trace a blue rhombus, turn as above, trace repeatedly to form a wheel?
At lower left, trace a red trapezoid, flip as above; check for reflection^T in the geomirror?
Write in the lower right quadrant? Repeat slides, flips, and turns with three other shapes on another grid?

^TNote: In math language, these transformations are called translations, rotations, and reflections.

^Swww.didax.com 21091J [pattern blocks] , 2-516J [geomirror] NCTM-G3- ©2013 MLA

Plane Shapes: Reflections in the Geomirror^T Lime 3.1.1PB1c



geomirror



Two friends will need pattern blocks, sheets of inch grid paper, and geomirrors.^T

each student, put a geomirror across a grid sheet, a block in front on your side. Is the mirror image the same as the block? Different?

Trace the block and its image!! Draw a line segment or a big "H" across the grid to represent the geomirror?

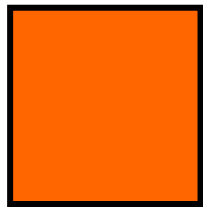
With a teacher, repeat for all the block shapes. How do they change in reflection?^T

Cut away excess paper; mount for display; write?^T

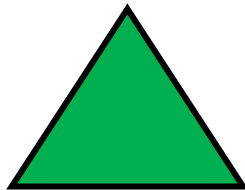
T-Note: Slides, turns, and flips can also be called translations, rotations, and reflections. A flip, the yellow hexagon looks the same but the green equilateral triangle may seem to "point" in opposite directions. We keep several geomirrors available in a labeled tote box. ^Swww.didax.com 210913J [pattern blocks], 2-516J [geomirror] NCTM G 2 ©2013 MLA

Plane Shapes: Meet the Pattern Blocks!^T

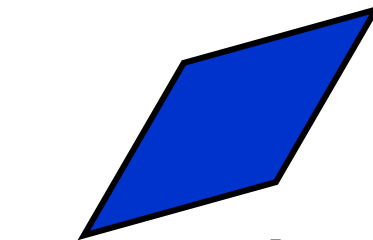
Lime 3.1.1PBl a



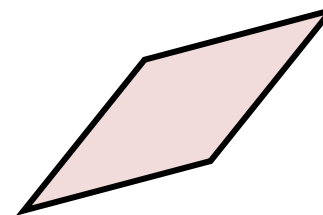
square



equilateral triangle



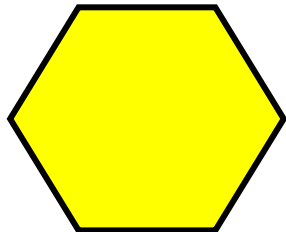
wide blue rhombus^T
or diamond



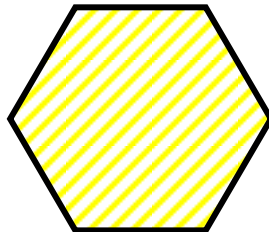
narrow tan rhombus or
diamond



trapezoid



hexagon



hexagon with hatching

The six pattern block shapes have faces^T that are polygons or plane (flat) shapes with straight sides.



Two friends will need: the pattern blocks,^{T S} colored pencils, journals, the pattern block template?^{T S}

Both students, trace and label one of each block—with the help of a template?

With a teacher, try some hatching; fill your paper with design; write.^T

^T-Note: The purpose is to explore the attributes (shapes and dimensions) of the pattern blocks. The pattern blocks tessellate—They fit together to fill two-dimensional (2D) space with no gaps and no overlaps. So we encourage students to fill out their designs. The block faces are plane (2D) shapes but the thickness makes the blocks solid shapes—3D. Most students can trace the blocks but, for some, the template helps by guiding the pencil. Templates are best stored in a wall file or teacher's tote. Webster's pronounces template with the schwa rather than the long a as in plate. The plural of rhombus is either rhombuses or rhombi. We prefer the latter classical Latin rhombi ending with the long “ee” sound. Hatching (a Montessorian practice) teaches focus.

Plane Shapes: Polygons, Eight Kinds^T

Lime 3.1.1GBb

Polygons are plane (flat) shapes with straight sides:

A trapezoid is a quadrilateral with two pairs of equal sides--one pair parallel.

A rhombus has four equal sides.

An isosceles triangle has three straight sides and two equal sides.

A parallelogram is a quadrilateral with two pairs of equal, opposite, parallel sides.

A hexagon has six equal sides.

An equilateral triangle has three equal sides.

A square is a rectangle with four equal sides and square corners.

A rectangle is a parallelogram with square corners.^T

Each student will need: square, circular, and isometric geoboards^S and dot matrices,^T colored elastics and matching pencils.^{T S}

Model a square on the square geoboard in colored elastic; draw on the square matrix in matching color; label as a square. Repeat for a larger square? Check for same number of dots and pegs, unit side lengths, and inside spaces in your drawings?

With a teacher and the same geoboards, repeat all for the other polygons with different colors — checking for part inside spaces too?^T Any that cannot be modeled?

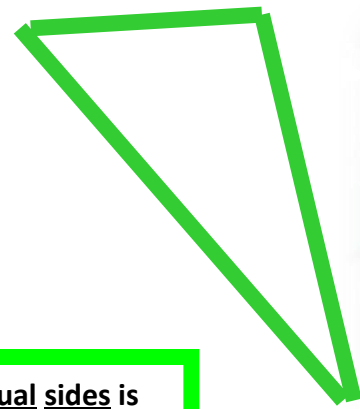
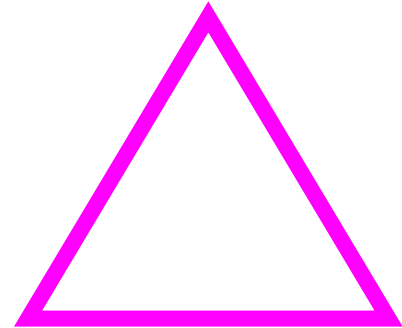
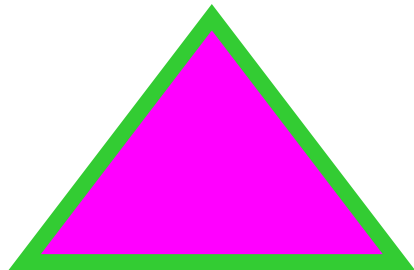
Repeat all with the isometric and circular geoboards.

Discuss, write, display?

^T-Note: The equilateral triangle, rhombus, and hexagon cannot be formed on a square geoboard.^S www.didax.com 2-420J, 210927J [geoboards] NCTM G 1 ©2013 MLA

Plane Shapes: Three Kinds of Triangles

Lime 3.1.1GBa



A triangle with two equal sides is an isosceles triangle.

A triangle with three equal sides is an equilateral triangle.

A triangle with three unequal sides is a scalene triangle.

Two friends will need: two square, 25-peg geoboards, two isometric geoboards, dot matrix paper to match both,^T six elastics of three different colors —colored pencils to match?

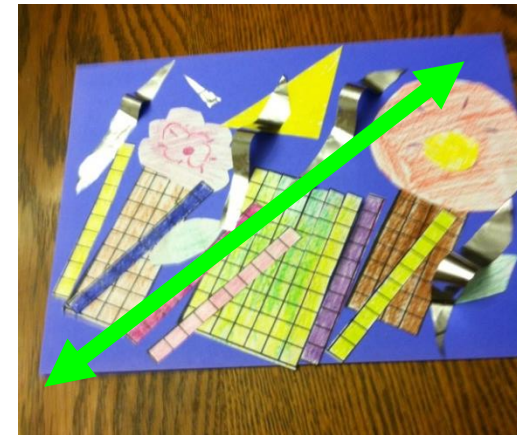
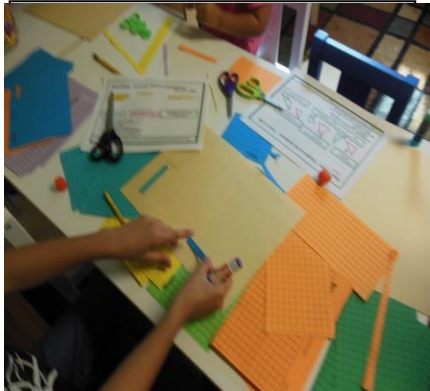
Both students, try forming the three types of triangles on the square geoboard. There will be one that cannot be formed. Which one?^T

With a teacher, draw on dot matrix in colored pencil; write. Repeat with a an isometric geoboard and isometric dot matrix?

^T-Note: The purpose is to see the possibilities and constraints of these matrices. The equilateral triangle cannot be formed on the square array geoboard because two sides would have to be longer or shorter than the pegs allow. The isometric geoboard will allow it. ⁵www.didax.com 2-420J, 2109271J [geoboards] SM 15.3.1DM, 15.3.4 NCTM G1 ©2013 MLA

Plane Shapes: Horizontal, Vertical & Perpendicular

Line 3.1.1C&G



Horizontal means across.

Vertical means up and down.

Perpendicular means crossing, with all corners the same size.

Two friends need: drawing paper, small pieces^T of cm grid of different colors, scissors, and glue sticks.

Cut strips of grid one cm wide and others two or three cm wide--all several cm long.

Array some onto drawing paper horizontally and some vertically—Are the strips now also perpendicular?

With a teacher, label and write for an artistic display? Add some strips of fancy paper? Glue everything in place.

A diagonal crosses your paper from corner to corner.

T-Note: The purpose is to establish meaning for these three terms common throughout geometry and measurement. Perpendicular can be defined as two line segments or shapes at right or 90° angles to one another. However, not all students will know how to check with a protractor. So, for this activity, we define perpendicular in terms of forming square corners. Students may enjoy adding metallic or textured strips of paper, cloth, or ribbon? We encourage filling in space in art work. We keep glue to a minimum—works best if weighted overnight! NCTM G--2 ©2013 MLA

Plane Shapes: Parallels, Intersects & Curves Line 3.1.1ASe

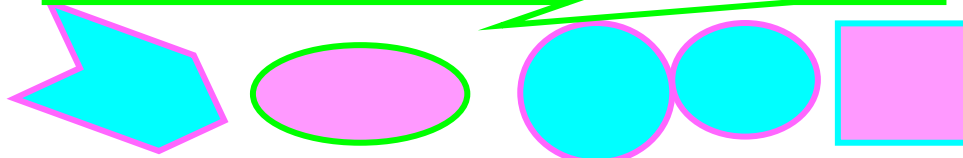
Straight lines have no angles (corners) or bends in them. We show them with arrowheads that mean they go on to infinity (∞). There are two kinds .

Parallel straight lines extend to ∞ (infinity) but never intersect (cross).^T

Intersecting straight lines do cross each other, but only once in all of ∞ .

A curve is any path (mark) on a plane (flat) surface that is drawn without lifting the pencil!^T It doesn't have to be straight! It can have bends and corners.

If the curve meets itself, it forms a closed curve.
Circles, squares, and figure eights are closed curves!!



Two friends will need: drawing paper and a straightedge.^T

With a straightedge, represent (draw):

two parallel lines;

two that intersect;

two more that curve.

Put arrowheads at each end to show that you're

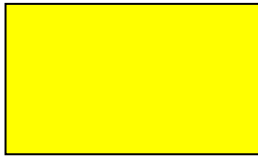
representing lines that go on to ∞ .

With a teacher, check the parallel lines with a ruler; label all.

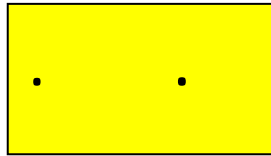
Draw some closed curves; fill in areas with color.

Plane Shapes: Points, Line Segments, & Polygons

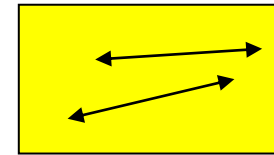
Line 3.1.1ASc



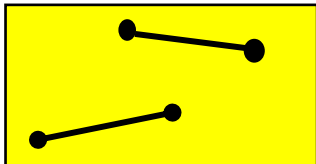
Points are invisible places in space.



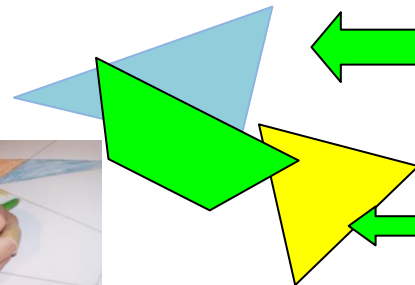
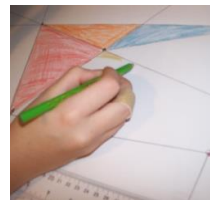
But we can represent them with pencil.



Lines are invisible rows of points that go on to infinity (∞) both ways. We show them with pencil.



Line segments are parts of invisible lines. The dots show their endpoints.



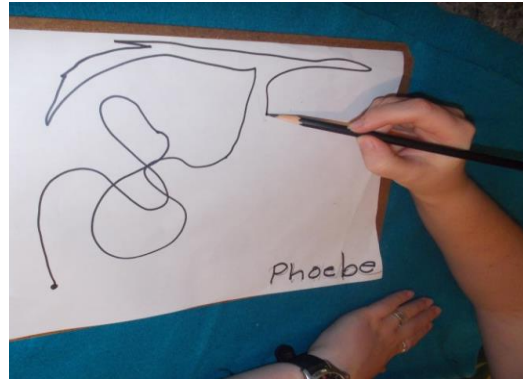
Three or more straight line segments connected form closed plane shapes called polygons.

This polygon is also a triangle.

Two friends will need: a straightedge,^T pencils, colored pencils, and drawing paper. Represent some points; connect for line segments;^T connect segments for polygons. With a teacher, extend segments to the paper's edges--imagine infinity!^T Color.^T

T-Note: Polygons too are really invisible because they have no thickness; as we add thickness with the pencil's graphite, we have constructed a three-dimensional (3D) or solid shape!! The straightedge can be any edge of a book or like object, but the ruler works best! Glitter glue suggests infinity! We insist that all or most of the drawing paper be filled with shapes and color. For safety metric rulers, see Didax.com or School Specialty.com.

Plane Shapes: Draw Curves & Closed Curves Lime 3.1.1ASb



A curve is a path that can be drawn in a plane, or flat, space without lifting a pencil.^T
A closed plane curve is a curve that begins and ends at the same point.^T
Artists combine curves and closed curves for interesting designs and

Two friends will need: two sheets of manila or white drawing paper,^T art pencils that leave a broad black mark, a clipboard, and fine markers?^T

Each student,^T put an art pencil at a point on drawing paper; move it along an interesting curve; turn so that you intersect with your own curve, forming a closed curve.

Without lifting pencil from paper, continue forming curves and closed curves to fill your paper.

With a teacher, look for trees, plants, buildings, people, or animals? Color the smallest closed curves and then some slightly larger ones until you're satisfied with your design?

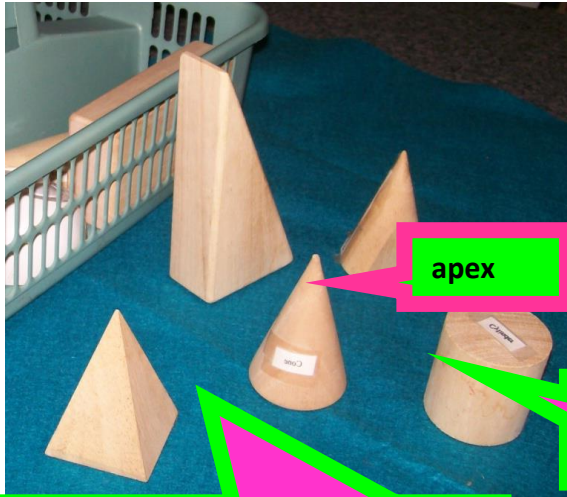
^T-Note: These definitions are taken directly from Balka, Bana, Hoover, Marshall & Swan, *Visual Math Dictionary* (15,56), a wonderfully clear resource—See *Bibliographies*. Consider good quality paper for sharp definition of line and color. We keep felt markers apart from students except for special projects and offer only the fine, slender ones--Their vivid colors and inkiness can distract from the purpose of the work

Section 3

Solid Shapes

Solid Shapes: Airplanes of Cylinders & Cones

Lime 3.1.2WGSb



apex

A cylinder has two circle bases.

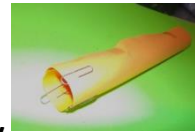
A cone has one circle as a base and an apex.

Two friends will need: the wooden geometric solids,ST copy paper, tape, paper clips, meterstick, long space, teacher with a camera, chalk of white, pink, and blue.

Younger student, roll a long side of paper to the opposite side; loosen the roll at one end and tighten it at the other for a “nose” of about a half cm across; apply a bit of tape—a truncated, oblique cone.

Older student, roll a cylinder; tape, with both ends the same.

Both students, with a teacher, mark off five meters in white chalk in the long space. Taking turns, fly your planes, marking landing places in different colors. Repeat, exchanging planes? Record differences; add paper clips in the noses? Record? Photograph each other flying. Write.



cylinder airplane



truncated or cut off

oblique or tilted

T-Note: Research slightly different designs on the Internet and in books? We keep the wooden solid shapes always within view.

^Swww.didax.com 2-450E [WGSs]

NCTM—2,4

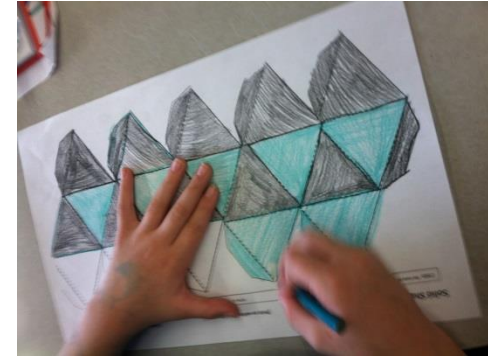
©2013 MLA

Solid Shapes: Construct an Icosahedron.

Lime 3.1.2SMh



Hexahedron Tetrahedron Octahedron Dodecahedron Icosahedron



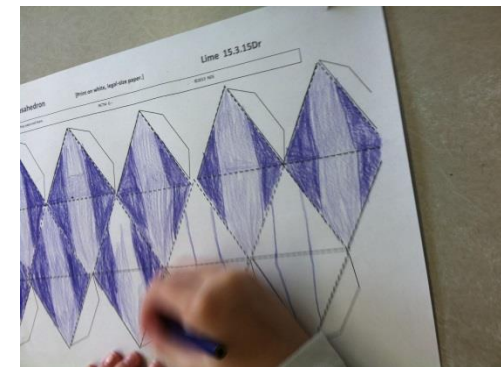
There are five regular polyhedra, also called Platonic solids, after Plato, an ancient Greek thinker.

An icosahedron (eye-CO-sah-hee-dren) is a regular polyhedron because its twenty sides are all the same.

Two friends will need: card stock net templates for the icosahedron, colored or drawing paper (to hatch or decorate), and binder clips.

Trace the net onto the paper; enter broken line segments in pencil; hatch or decorate? Cut out the net on the unbroken line segments; fold away from yourself on the broken segments; glue the tabs; assemble; clamp with the clips to dry overnight?

With a teacher, write and prepare for a display?



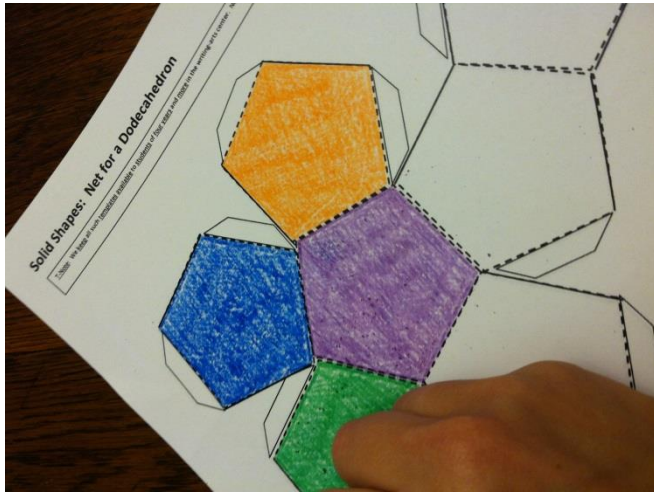
T-Note: The *SM 15.3.15Dr* is meant to be turned into a card stock template by teachers. Students may need help in penciling in the broken fold line segments. We collect student solid shapes and their associated writing onto a display with colored burlap for the background.

SM 15.3.15Dr

NCTM G-- 1

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Solid Shapes: Construct a Dodecahedron Lime 3.1.2SMg



The dodecahedron (doh-deck-a-hee-dren) is a regular polyhedron because its twelve polygonal sides are all the same five-sided pentagons. The sides of the pentagons are edges to the dodecahedron.

Two friends will need: two card stock net templates for the dodecahedron, colored or drawing paper (to be hatched or decorated, and binder clips.

Trace the net onto the paper; enter broken line segments in pencil; and hatch or decorate? Cut out the net on the unbroken line segments; fold away from yourself on the broken segments; glue the tabs; assemble; clamp with clips to dry overnight?

With a teacher, write and prepare for a display?

T-Note: The dodecahedron is a regular polyhedron and considered one of the five Platonic solids. The SM 15.3.14Dr is meant to be turned into a card stock template by teachers. Students may need help in penciling in the broken fold line segments. We collect student solid shapes and their associated writing onto a display with colored burlap for background. NCTM G--1 ©2013 MLA

Solid Shapes: Construct an Octahedron.

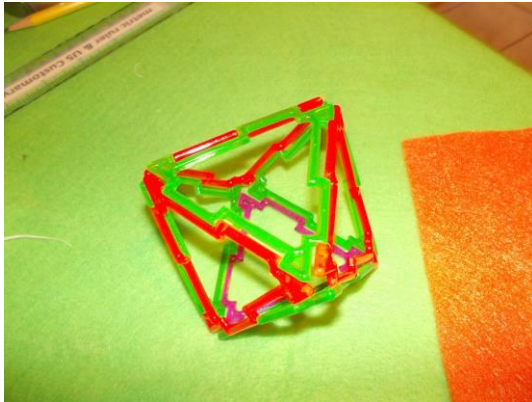
Lime 3.1.2SMf

An octahedron is a polyhedron because its eight faces are polygons.

The prefix "octa-" means eight.

It is a regular polyhedron because all of its faces are the same equilateral triangles.

Regular polyhedra are also called Platonic solids.



Materials for two friends: copies of the *Net for an Octahedron* from *Spectrum Masters*, glue sticks, and binder clips.

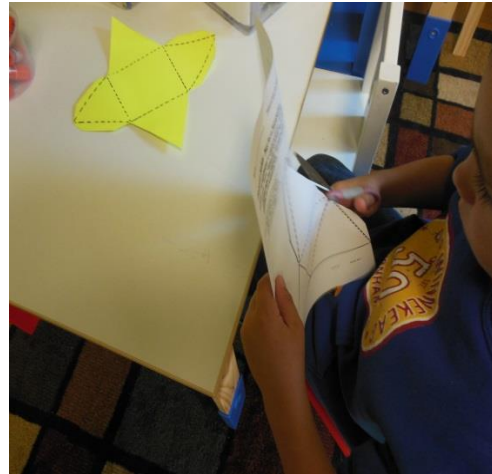
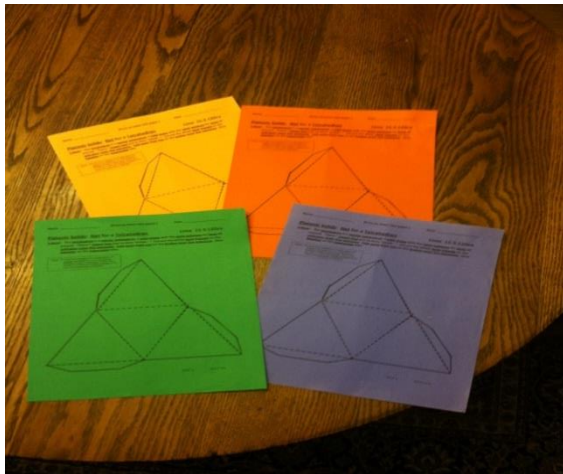
Decorate your octahedra? Cut the nets along the outside unbroken line segments; fold away from yourself along the inside broken line segments; glue the tabs; pinch the edges with clips? Dry overnight in the clips?

With a teacher, write and display?

T-Note: Students feel grown-up in learning this Greek vocabulary and engineering something 3D! We keep a set of solid geometric shapes labeled and displayed some assembled from recycled containers. This regular polyhedron is considered one of the five Platonic solids. ⁵www.didax.com 2-450H [wooden geometric shapes] SM 15.3.13Dra or b NCTM G 2 ©2013 LMA

Solid Shapes: Construct a Tetrahedron.

Lime 3.1.2SMe



A tetrahedron is a polyhedron because all of its faces are polygons.

It is a regular polyhedron, because all of its four faces are the same equilateral triangles.

Another name for a regular polyhedron is Platonic solid.

Materials for two friends: copies of the *Net for a Tetrahedron* from *Spectrum Masters*, glue sticks, and binder or paper clips.

With a teacher, cut the nets along the outside unbroken line segments; fold away from yourself along the inside with clips? Dry overnight in the clips? Write and display?

T-Note: Students feel grown-up in learning this Greek vocabulary and engineering something 3D! We keep a set of solid geometric shapes labeled and displayed--some assembled from recycled containers. This regular polyhedron is considered one of the five Platonic solids. ⁵www.didax.com 2-450H [wooden geometric shapes] SM 15.3.12Dra or b NCTM G 2 ©2013 MLA



Solid Shapes: Fold a Square Pyramid.^P

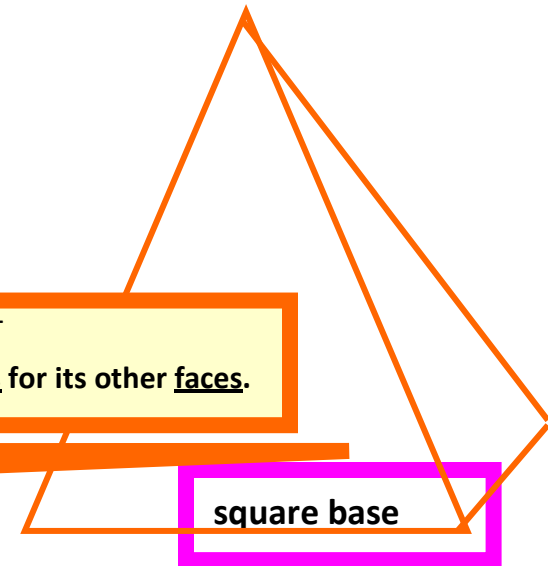
Lime 3.1.2SMd



Polyhedra are solid shapes^T with polygons as faces, forming straight edges.^T

A pyramid is a polyhedron with one polygon for a base and triangles for its other faces.

The base of this pyramid is square, so it's a square pyramid.



the Great Materials for two friends: copies of the *Net for a Square Pyramid*, glue sticks, and binder or paper clips.

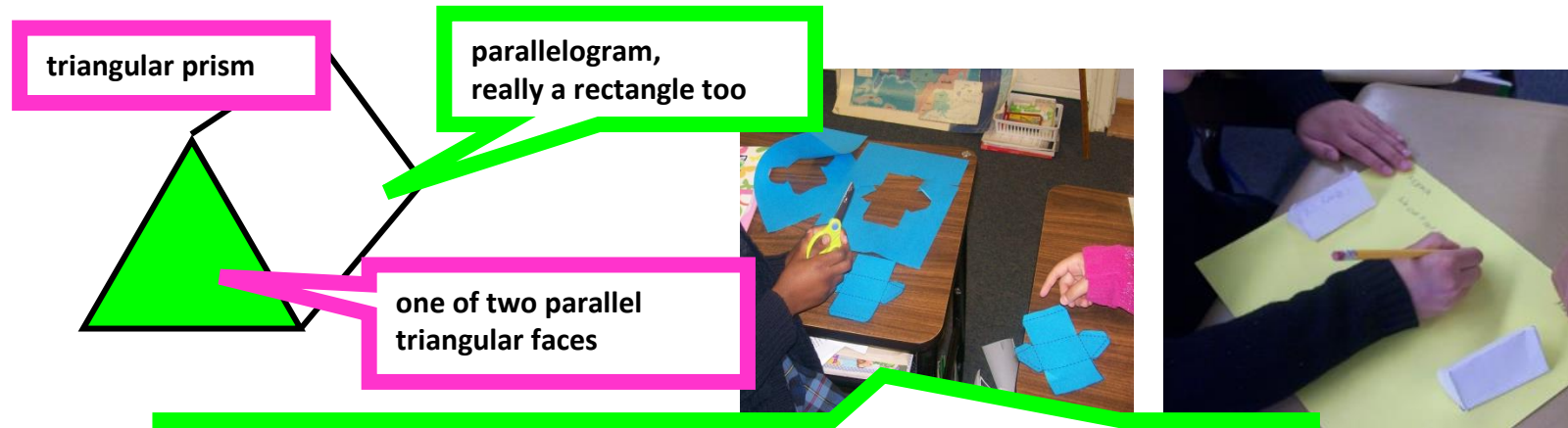
With a teacher, cut the nets along the outside unbroken line segments; fold away from yourself along the inside broken line segments; glue the tabs; pinch the edges with clips? Dry overnight in the clips?

Write and display?

Research Pyramid of Giza; combine all in a display?

^{T-Note:} Plenty of discussion is constructive here, and in all the net-to-solid shape activities, in order to develop the interrelated, geometric vocabulary. We keep a set of solid geometric shapes labeled and displayed--some assembled from recycled containers. Swww.didax.com 2-450H [wooden geometric shapes] SM 15.3.11Dra or b © Richard Seaman NCTM G 2 ©2013 MLA

Solid Shapes: What's a Triangular Prism? Lime 3.1.2SMc



We can make solid shapes like this triangular prism from plane shapes called nets.

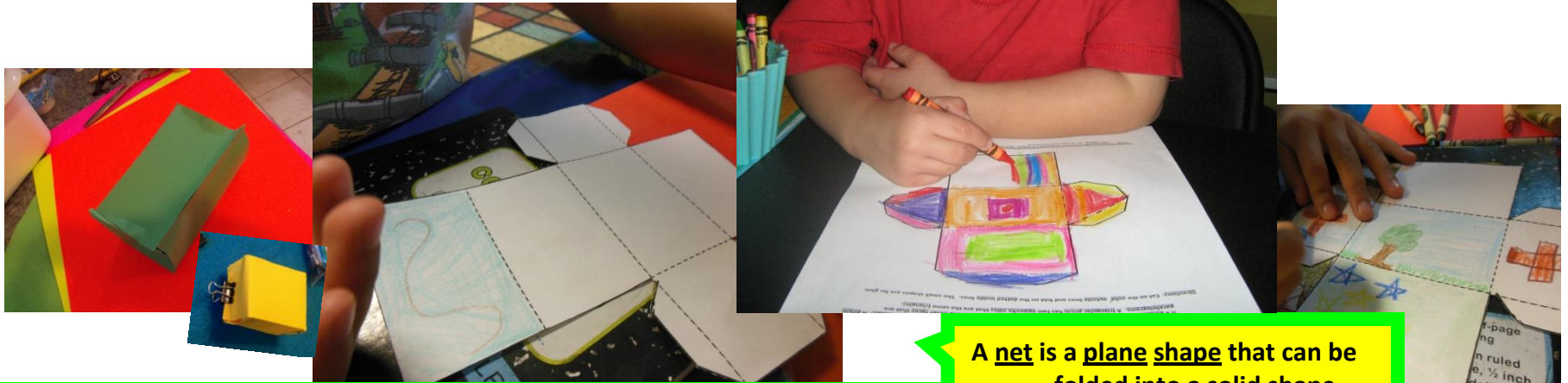
Polyhedra are solid shapes with polygons for faces.^T A prism is a polyhedron with two opposite faces the same and parallel like the triangles at the ends of this polyhedron. A prism must also have faces joined by parallelograms. So this is a triangular prism.

Materials for two friends: copies of Net for a Triangular Prism from Spectrum Masters, glue sticks, and binder or paper clips.

With a teacher, cut the nets along the outside, unbroken line segments; fold away from you along the inside broken line segments; glue the trapezoidal tabs; pinch the edges with clips? Write about what you did, using all the new attribute words above? Leave the clips on overnight? Form a display?

T-Note: Students feel grown-up in learning this Greek vocabulary and engineering something 3D! We keep a set of solid geometric shapes labeled and displayed--some assembled from recycled containers. One Swiss candy comes in triangular prisms! The rectangles in the two Spectrum Masters approximate the "golden rectangle" ratio of width to length that is about 1.6.

Solid Shapes: What's a Rectangular Prism? Lime 3.1.2SMb



A net is a plane shape that can be folded into a solid shape.

A polyhedron is a solid shape, of length, width, and depth and polygonal faces.

A rectangular prism is a polyhedron has three pairs of faces that are parallel but opposite each other.

Two friends will need: copies of *Net for a Rectangular Prism* from *Spectrum Masters*,^T scissors, glue sticks, binder clips or paper clips.^T

Decorate your net any way you like. Cut on the unbroken outside line segments; fold away from yourself on the inside broken line segments; glue the tabs; secure with clips to dry overnight?

With a teacher, write about what you did? Plan more polyhedra?

T-Note: Here is vocabulary development and feeling for real 3D forms. The dimensions of length, width, and depth can be applied interchangeably. We encourage careful consideration of all the attributes (characteristics) of the solids. These projects can be mounted onto interesting backgrounds like colored burlap, with student writings, for really attractive displays that teach! The rectangles in the nets are "golden rectangles" in the "golden ratio" of ϕ —meaning that one dimension divided by the other approximates 1.6. SM 15.3.9Dra or b NCTM G--1 ©2013 MLA

Solid Shapes: Fold a Cube.

Lime 3.1.2SMa



Solid shapes have three dimensions of length, width, and depth!^T These solids are **polyhedra** because they have **polygons** for sides.

Polyhedra can be formed from plane, or two-dimensional shapes, called nets.

This **net** can make a **cube**, a **regular polyhedron**, because all its six **polygon faces** are the **same**.

Each student will need: a **Net for a Cube**,^T glue sticks, and **binder clips**.^T
Cut on the **unbroken outside line segments**; **fold away from yourself on the inside broken line segments**; glue the **tabs**; **secure with clips**; leave to **dry overnight**?
With a **teacher**, write about your **regular polyhedron** and **how you made it**;^T **plan more constructions**? Display?

T-Note: Dimensions of length, width, and depth can be applied to solids as one sees fit—interchangeable terms! We reserve the term “solid” to describe solid shapes and use “unbroken” to describe “solid” line segments. We encourage discussion of all identifiable attributes (characteristics) of the solids. The cube also meets the requirements of the definition for a prism. The cube is one of five regular polyhedra called the “Platonic Solids.” These projects can become displays that teach!



Solid Shapes: Wasps & Bees at Home

Lime 3.1.2Res



Wasps and bees build their homes in the form of many hexagonal cylinders with two hexagonal faces and five rectangular sides.

Two friends will need: print and online research sources, tracing paper, pencils and gold or yellow colored pencils?

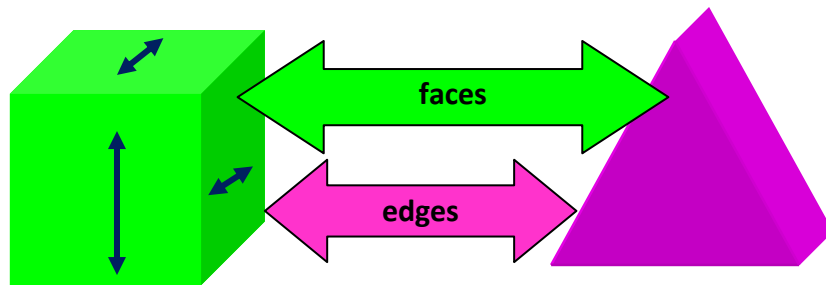
In both kinds of sources, find pictures of wasp and bee communities. Trace them from books? Print them from the Internet? Copy from the Internet?

Write about how these homes are built. Do all individuals build? Gather food? What else has to be done?

With a teacher, write about what you learned?

Solid Shapes: What Is & Is Not a Polyhedron? Lime 3.1.2GS

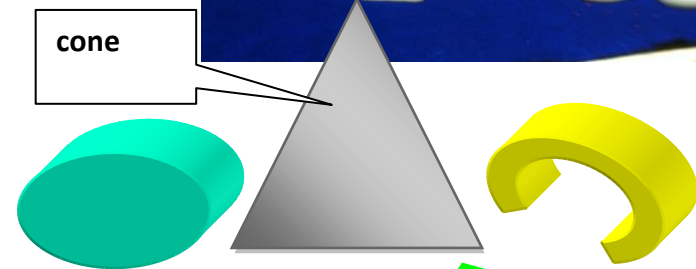
All solid shapes have three dimensions of height, width, and depth—3D, for short.^T



Polyhedra are solid shapes with polygons for faces and straight edges. An edge is where the sides of the polygons meet.

Materials for two friends: a set of solid shapes,^T drawing paper, art pencils, and a teacher with a camera, color printer, and, perhaps, a safe light source.^T

Make an array of solid shapes. Make a set of polyhedra and non-polyhedra. With a teacher, photograph your sets; print your photos in a size to fit your journals, label and write. You may want to try drawing the solids with the help of the table lamp to cast shadows!

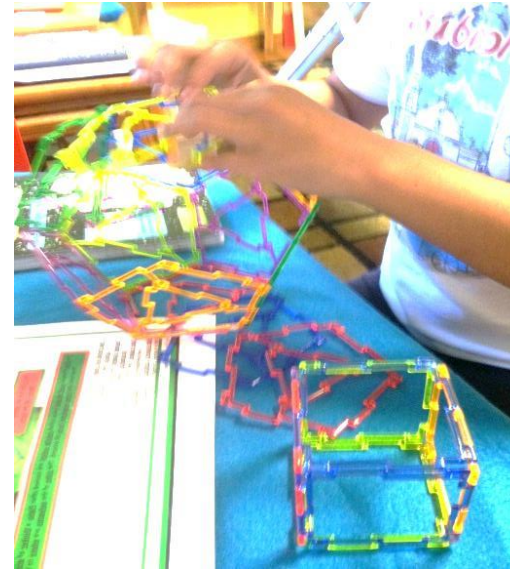


These solid shapes are not polyhedral because they have curved faces and edges.

^T-Note: Solid shapes can be purchased or made from nets found in *White Series*. Any table lamp (a “gooseneck”?) will show effects of light angles, with some positioning. Drawing pencils will “smudge” for shadow effects. We keep all in a tote box.^Swww.didax.com 2-450H [geometric solids] SM 15.3.8-.15 NCTM G 2 ©2013 MLA

Solid Shapes: Platonic Solids

Lime 3.1.2GFb



The five Platonic solid shapes (also called regular solids) are named after Plato, a thinker of ancient Greece.

They have the same polygonal faces all around. The sides of the polygons are edges to the polyhedra.

They are called the tetrahedron (four faces), cube (six faces, also called the hexahedron), octahedron (eight faces), dodecahedron (twelve faces), and icosahedrons (twenty faces).

Two friends will need: the Geofix® set,⁵ a teacher with a camera, and a color printer.

Younger student, locate pieces in the set that correspond to the faces of the tetrahedron and the cube; snap them together to form two Platonic solids.

Older student, take a turn with the octahedron and dodecahedron.

Both students, construct the icosahedron together.

With a teacher, photograph, print, and write.

T-Note: The Geofix® set has plane shapes to assemble into many different solids; we encourage investigation of this material to see how many solids can be formed. These constructed shapes are called solids even though transparent! We print four or more photos per page to save on the color ink.

⁵www.didax.com 3-205J or 210885W [Geofix® explorations set]

NCTM—G1

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Solid Shapes: Build the Platonic Solids.

Lime 3.1.2GFa

The dodecahedron has twelve pentagonal faces—all the same.

The cube has six square faces—all the same.

The octahedron has eight equilateral triangles—all the same.

The icosahedron has twenty!

The tetrahedron has four.

The five Platonic solid shapes can also be called regular solids or regular polyhedra. They have the same polygonal faces all around. The sides of the polygons are edges to the polyhedra. They are named after Plato, a thinker of ancient Greece.

Two friends will need: the Geofix® set,^s a teacher with a camera and printer.

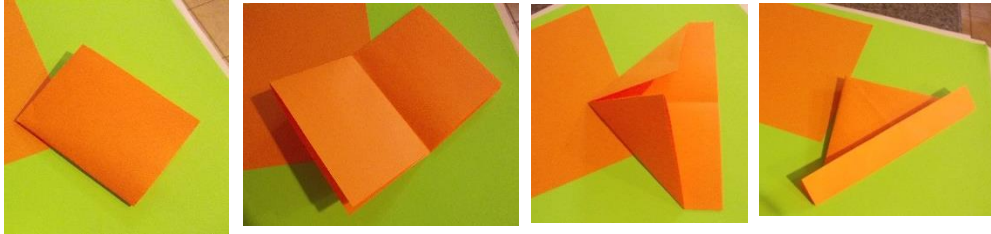
Younger student, assemble four triangles into a tetrahedron;
repeat with six squares for a cube.

Older student, form eight triangles into an octahedron
and twelve pentagons into a dodecahedron.

Both students, construct the icosahedron from twenty triangles.

With a teacher, photograph, print, and write.

Solid Shapes: Fold a paper Boat



Two friends will need: sheets of colored copy paper and a straightedge for a sharp creaser?

Both students, holding the paper vertically, fold in half top to bottom; fold in half again left to right and unfold; fold left and right corners down to the center, crease.

Fold the bottom strip upwards; turn over and repeat;^T open between these latter folds, forming a rhombus; fold the bottom angled corner up to the top one; turn over and repeat; open the triangle from the bottom and flatten for a new rhombus, pick it up, pull it apart at the top angled corners to find the boat; pull it into a canoe with a cabin?

Lime 3.1.2FM



Section 4

Black Line Masters

Solid Shapes: Net for an Icosahedron

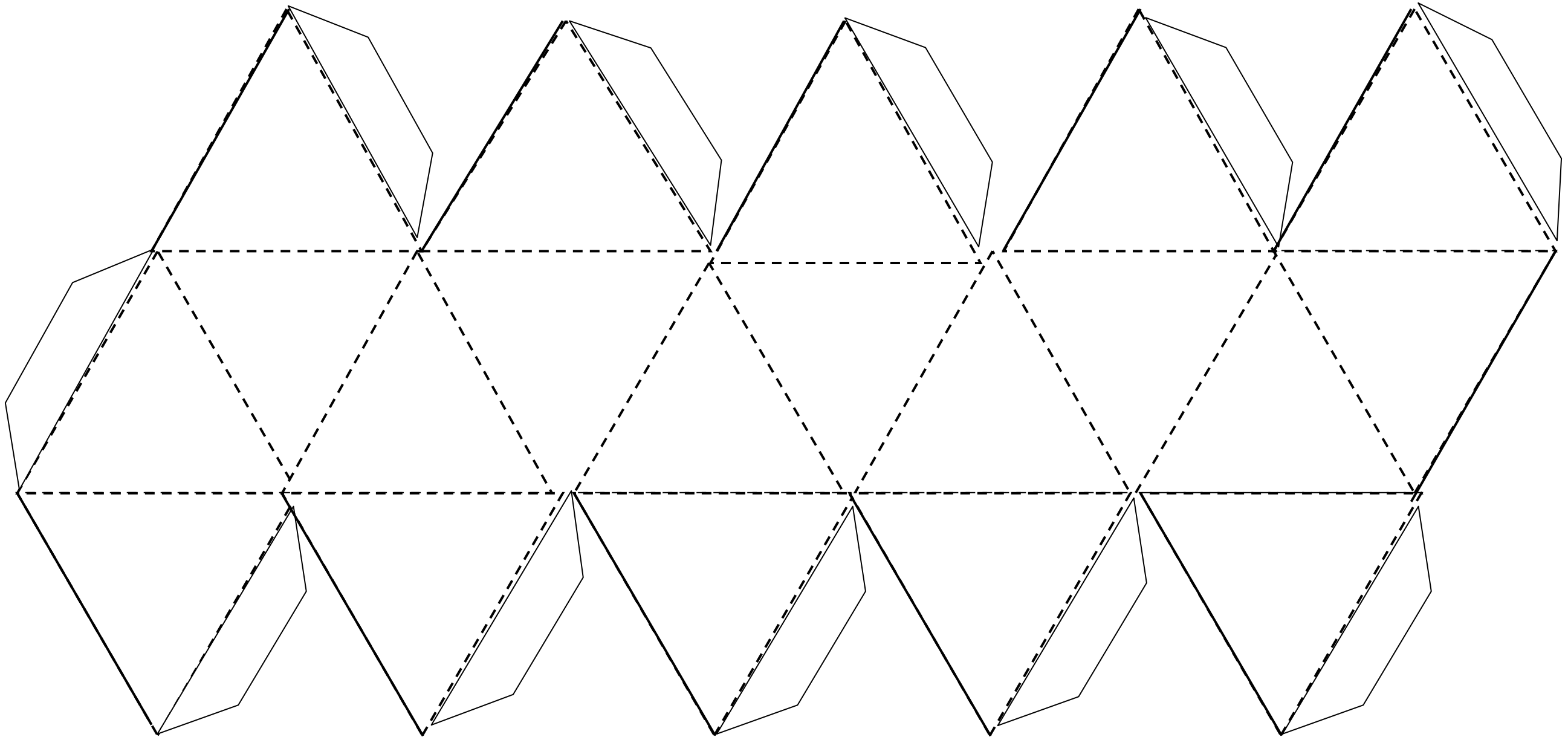
[Print on white, legal-size paper.]

Lime 15.3.15Dr

T-Note: We make nets available anytime to students of four years and more.

NCTM G, 1

©2013 MLA

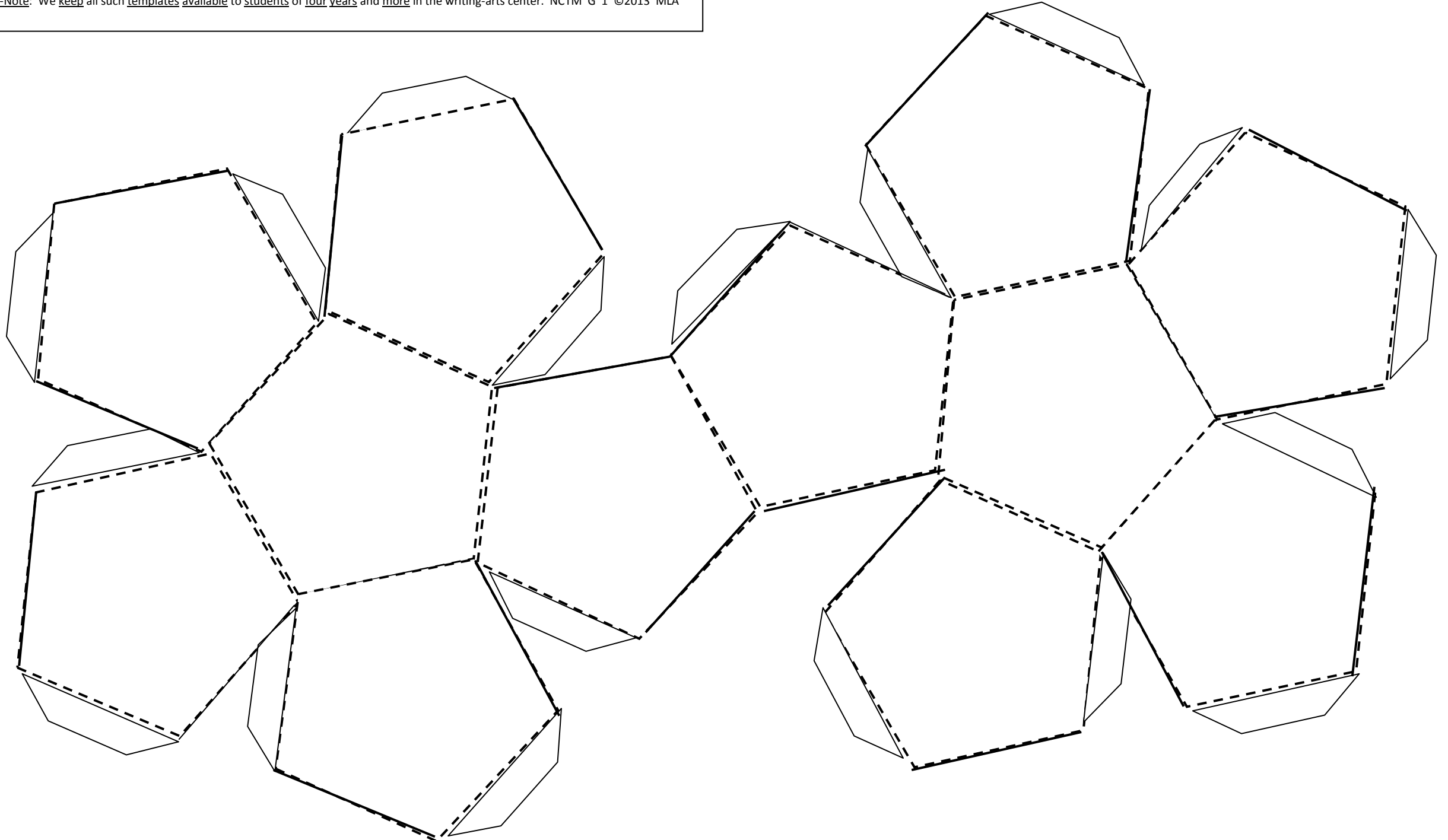


Solid Shapes: Net for a Dodecahedron

[Print on white, legal-size paper.]

Lime 15.3.14Dr

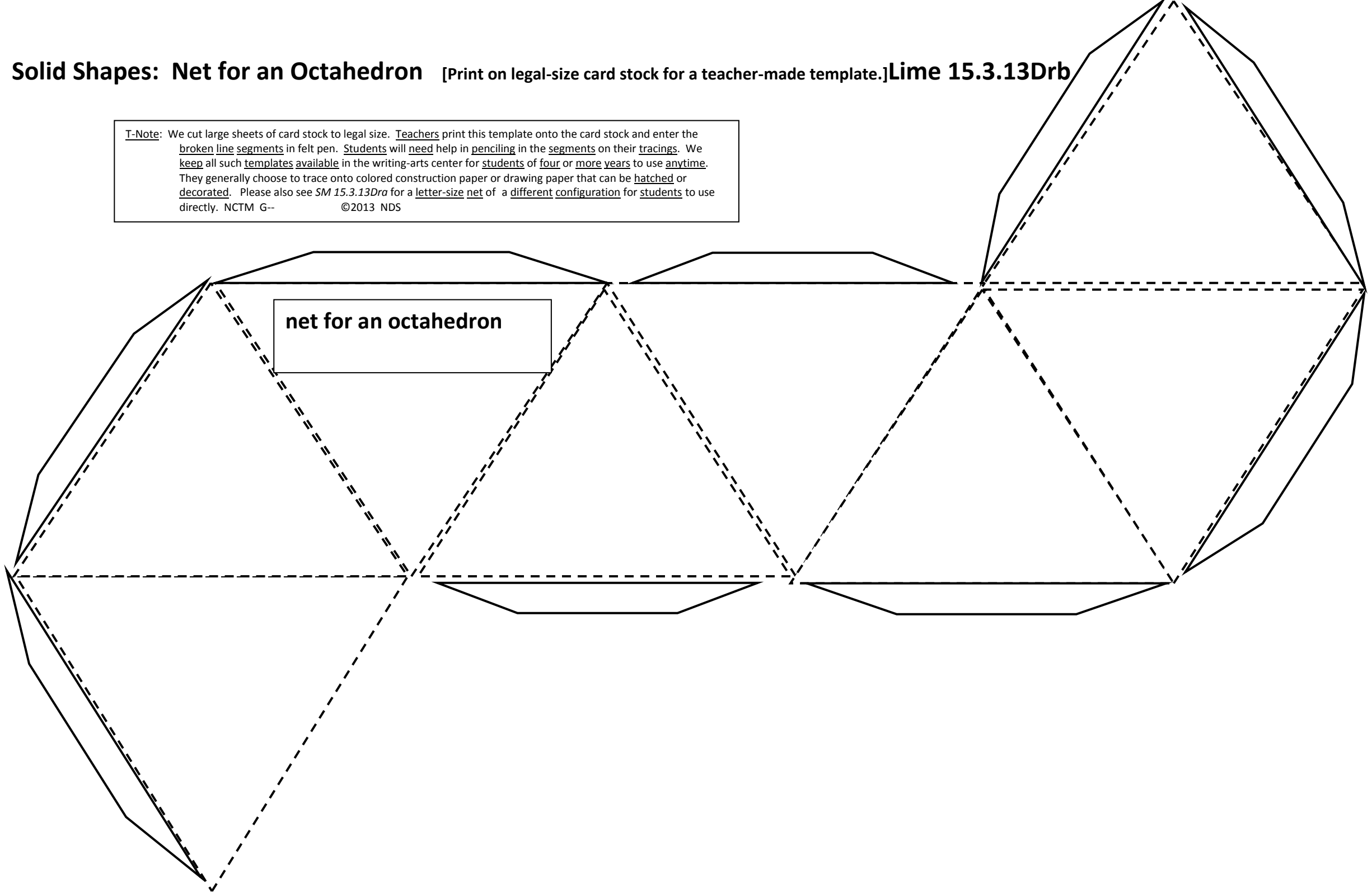
T-Note: We keep all such templates available to students of four years and more in the writing-arts center. NCTM G 1 ©2013 MLA



Solid Shapes: Net for an Octahedron [Print on legal-size card stock for a teacher-made template.]Lime 15.3.13Drb

T-Note: We cut large sheets of card stock to legal size. Teachers print this template onto the card stock and enter the broken line segments in felt pen. Students will need help in penciling in the segments on their tracings. We keep all such templates available in the writing-arts center for students of four or more years to use anytime. They generally choose to trace onto colored construction paper or drawing paper that can be hatched or decorated. Please also see *SM 15.3.13Dra* for a letter-size net of a different configuration for students to use directly. NCTM G-- ©2013 NDS

net for an octahedron



Name _____

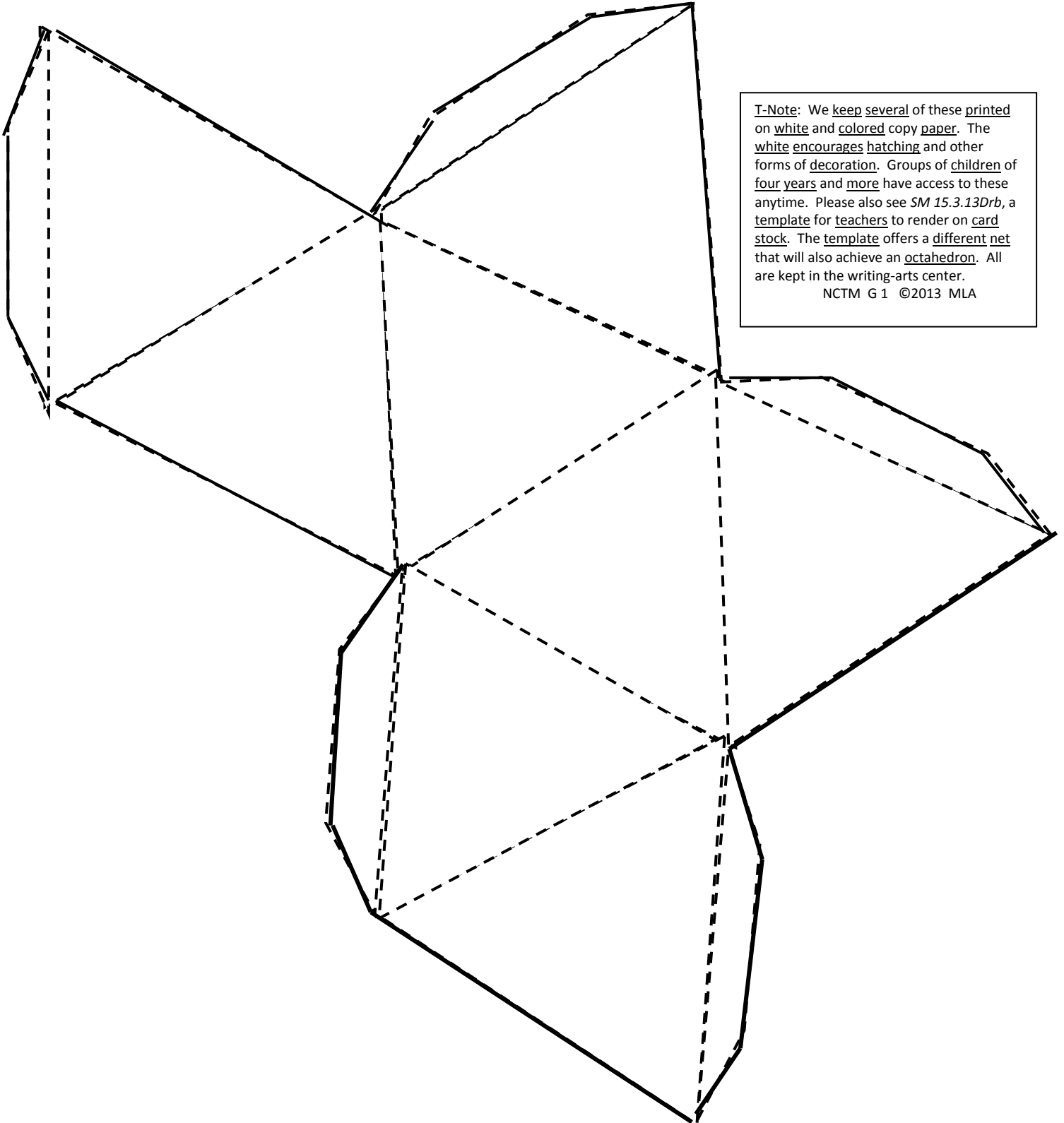
[Print on letter-size paper.]

Date _____

Net for an Octahedron

Lime 15.3.13Dra

S-Note: The octahedron is a regular polyhedron and a Platonic solid that means it has the same polygon with the same corners for all of its faces. “Octa” is a prefix that means eight. Cut on the unbroken outside line segments; fold on the inside broken line segments; always fold away from yourself.
Glue on the tabs.



T-Note: We keep several of these printed on white and colored copy paper. The white encourages hatching and other forms of decoration. Groups of children of four years and more have access to these anytime. Please also see SM 15.3.13Drb, a template for teachers to render on card stock. The template offers a different net that will also achieve an octahedron. All are kept in the writing-arts center.
NCTM G 1 ©2013 MLA

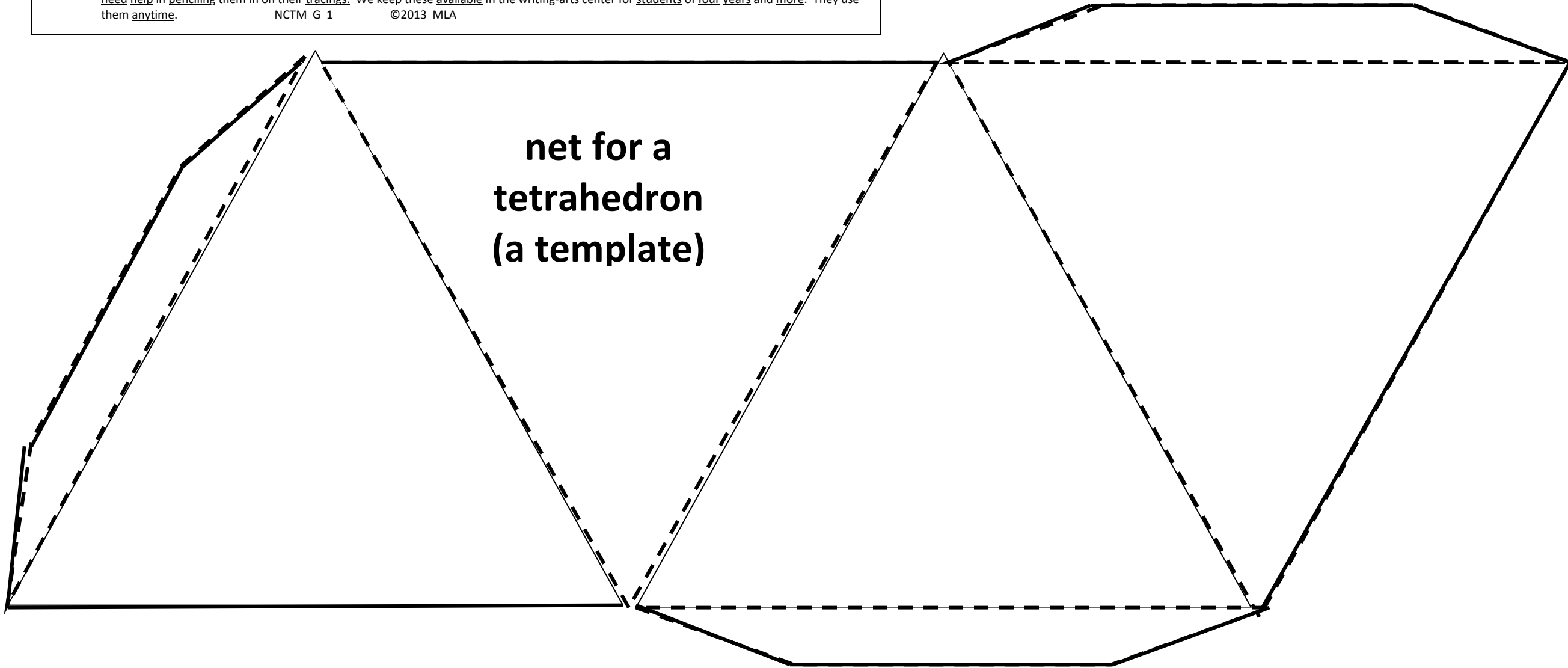
Solid Shapes: Net for a Tetrahedron

[Print on legal-size card stock or paper for a teacher-made template.]

Lime 15.3.12Drb

T-Note: This template is intended to be printed onto legal-size card stock by the teacher. Next, the teacher can assemble the template by cutting away the last triangle on the right and attaching it to the middle triangle with the glue tab on the right. The result will be a large triangle corresponding to the template in *SM 15.3.13Dra*—intended for direct use by students. The teacher can enter the broken line segments in felt pen. Students will need help in penciling them in on their tracings. We keep these available in the writing-arts center for students of four years and more. They use them anytime.
NCTM G 1 ©2013 MLA

**net for a
tetrahedron
(a template)**



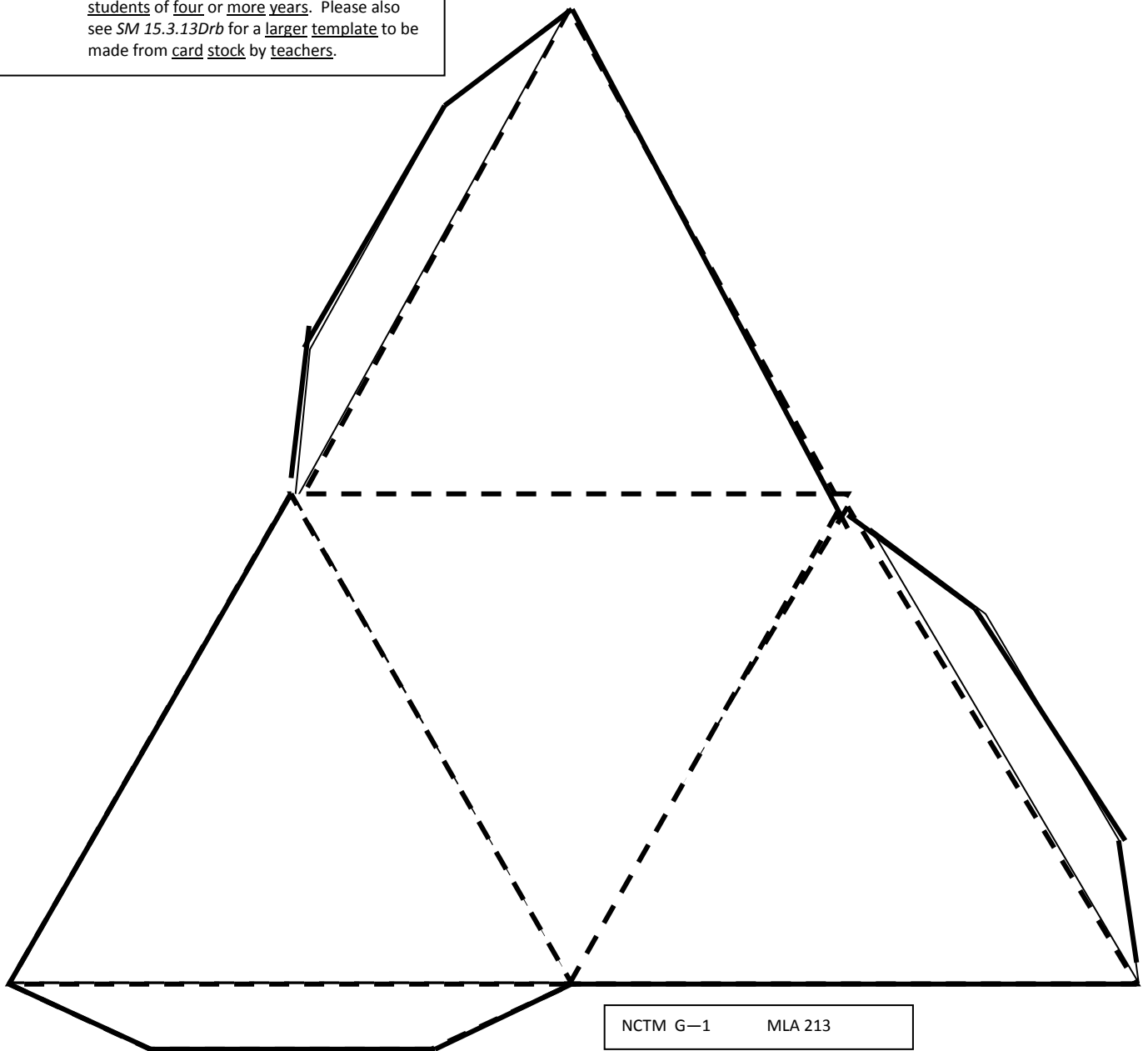
Name _____ [Print on letter-size paper.] Date _____

Platonic Solids: Net for a Tetrahedron

Lime 15.3.12Dra

S-Note: The tetrahedron is a regular polyhedron, a solid shape with the same polygons for faces all around. “Tetra “ means four just as does “quad-.” Cut out the whole large triangle, including the glue tabs, on the unbroken outer line segments. Fold away from you on the broken inner line segments. Glue together on the trapezoidal tabs. Allow to dry as long as possible.

T-Note: We keep these printed on both white and colored copy paper for use at any time by students of four or more years. Please also see SM 15.3.13Drb for a larger template to be made from card stock by teachers.

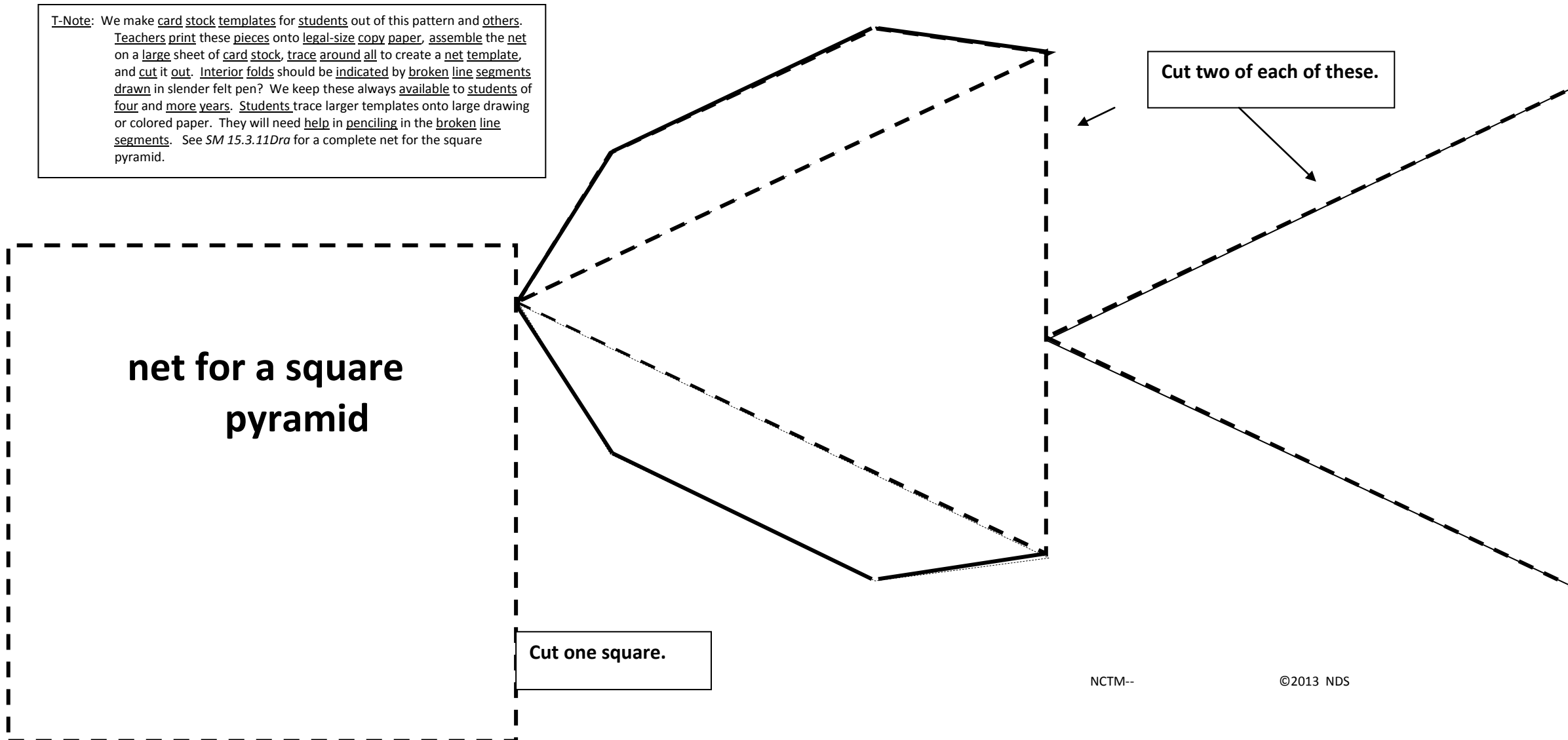


Solid Shapes: Net for a Square Pyramid^T

[Print on legal-size for teacher-made template.]

Lime 15.3.11Drb

T-Note: We make card stock templates for students out of this pattern and others. Teachers print these pieces onto legal-size copy paper, assemble the net on a large sheet of card stock, trace around all to create a net template, and cut it out. Interior folds should be indicated by broken line segments drawn in slender felt pen? We keep these always available to students of four and more years. Students trace larger templates onto large drawing or colored paper. They will need help in penciling in the broken line segments. See *SM 15.3.11Dra* for a complete net for the square pyramid.



Name _____ [Print on letter-size paper.]

Date _____

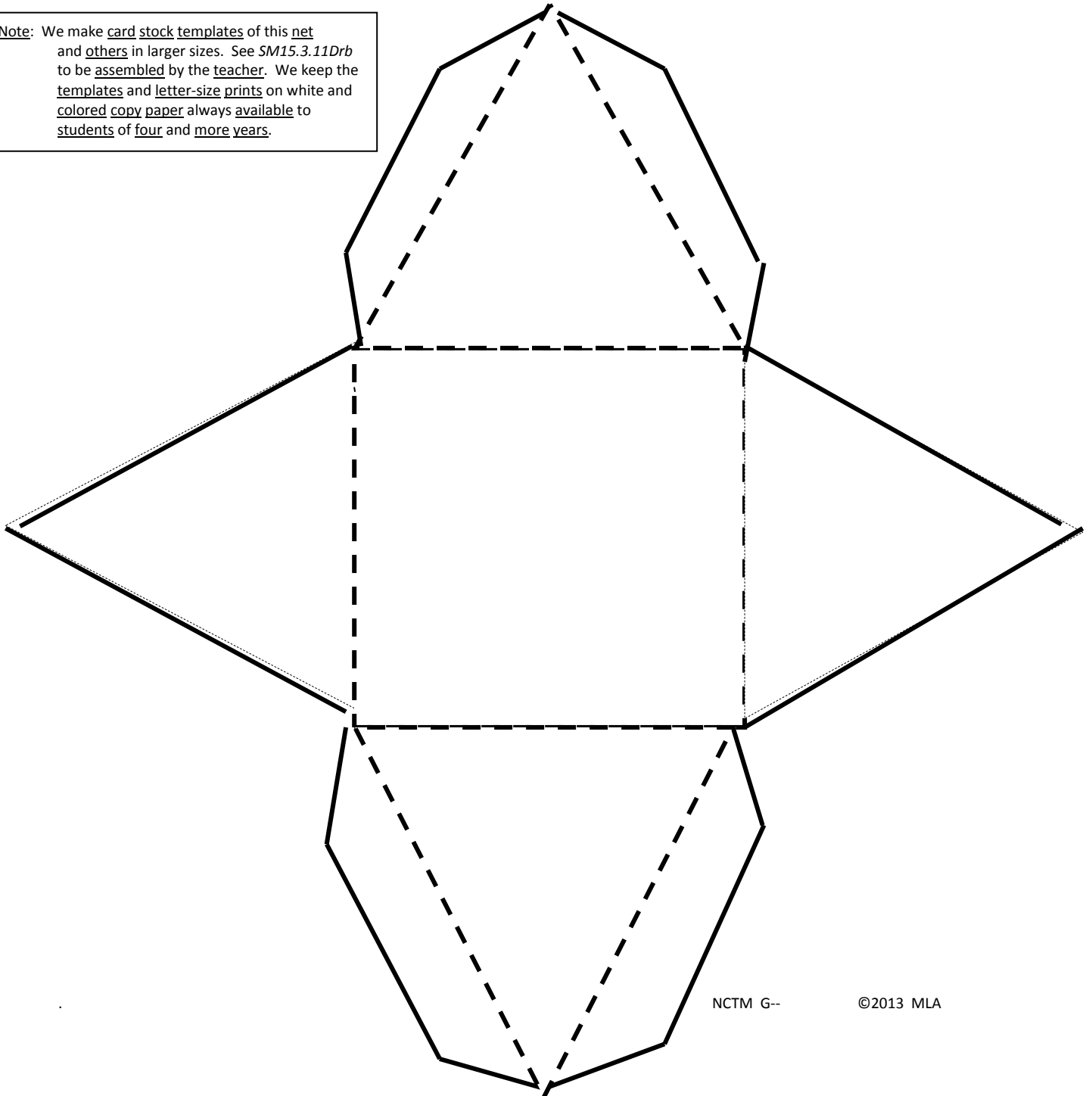
Net for a Square Pyramid^T

Lime 15.3.11Dra

S-Note: Cut on the unbroken outside line segments. Fold on the broken inside line segments.

With the square in front of you, fold the triangles away from you. With the top triangle in front of you, fold the two tabs away; repeat for the lower triangle. Glue on the trapezoidal tabs, assemble, and fasten with binder clips. Allow to dry for as long as possible.

T-Note: We make card stock templates of this net and others in larger sizes. See SM15.3.11Dra to be assembled by the teacher. We keep the templates and letter-size prints on white and colored copy paper always available to students of four and more years.



Name _____

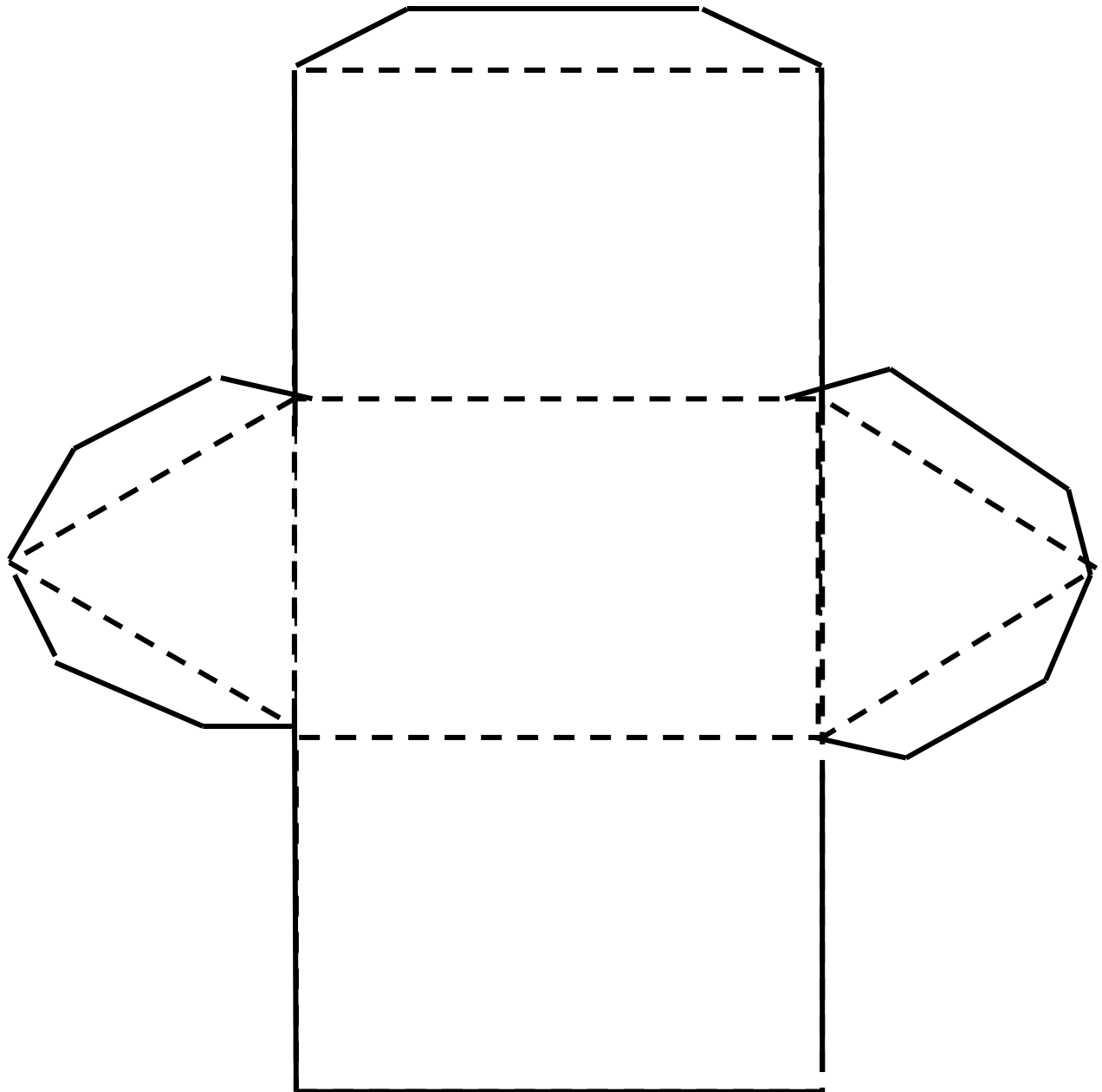
[Print on letter-size paper.]

Date _____

Net for a Triangular Prism

Lime 15.3.10Dra

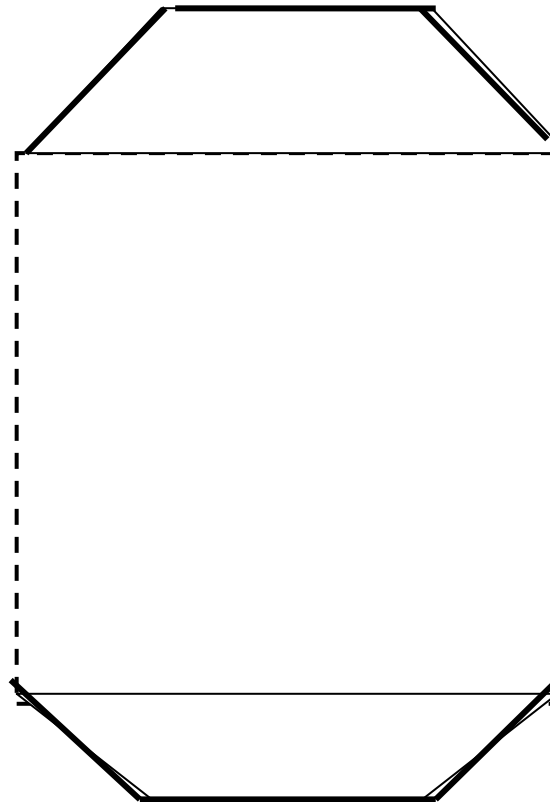
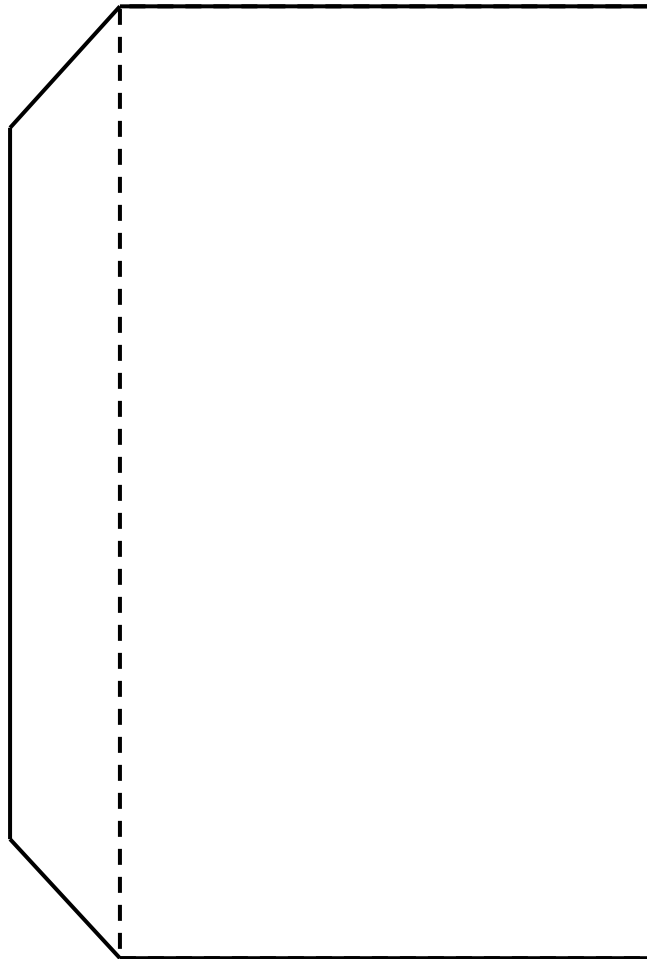
S-Note: Cut on the unbroken outside line segments; with the rectangles and triangles in front of you, fold away from you on all the broken line segments. Glue stick the trapezoidal tabs, assemble, and pinch with binder clips, allow to dry for as long as possible.



Solid Shapes: Net for a Rectangular Prism

[Print first on letter-size paper.]

Lime 15.3.9Drb



These two shapes can be reproduced for a net for a rectangular prism --larger than the one in *SM 15.3.9Dra*.

1. Print these two plane shapes on letter-size copy paper.
2. Trace onto a large sheet of card stock as a row of four of the rectangular shape--parallel with the long sides.
3. Add the long glue tab to one end of the row.
3. Trace the square and its glue tabs to both ends of the rectangle second from one end of the row.

(See *SM 15.3.9Dra* for a complete idea.)

4. Enter broken line segments in slender felt pen to show where folds are to be made
5. Label as "net for a Rectangular Prism."

Teacher-made templates reside in the writing-arts center for students to use anytime they have met other requirements. We make such materials available to students of four and more years.

Name _____ [Print on letter-size paper.]

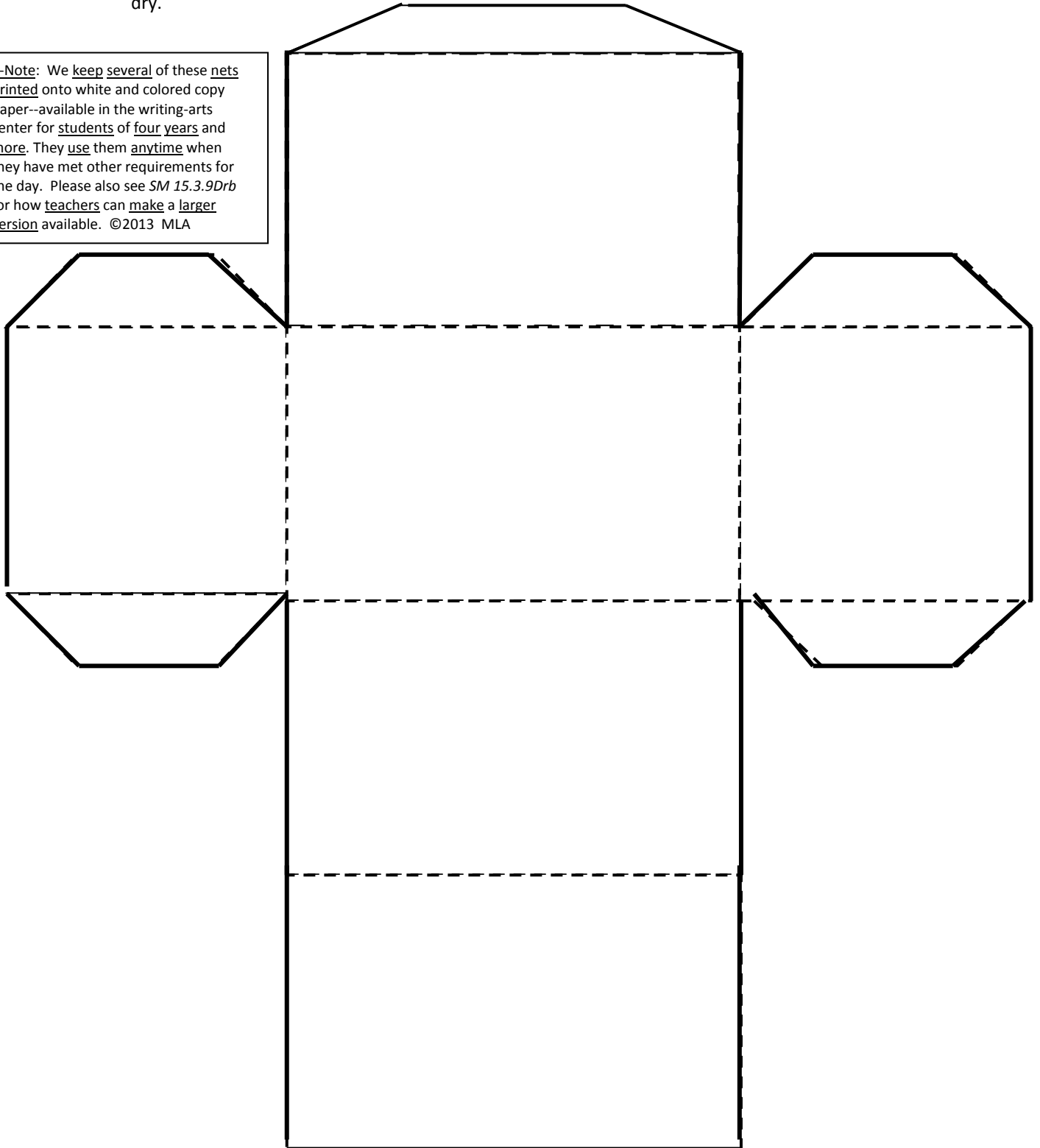
Date _____

Net for a Rectangular Prism^T

Lime 15.3.9Dra

S-Note: Cut on the outside line segments and fold on the broken inside line segments. The larger rectangles should be 8 cm by 5 cm or a “the golden rectangle.” Hold them in front of you and fold the squares and trapezoidal glue tabs away from you. Glue stick the tabs, assemble, pinch with binder clips, and allow to dry.

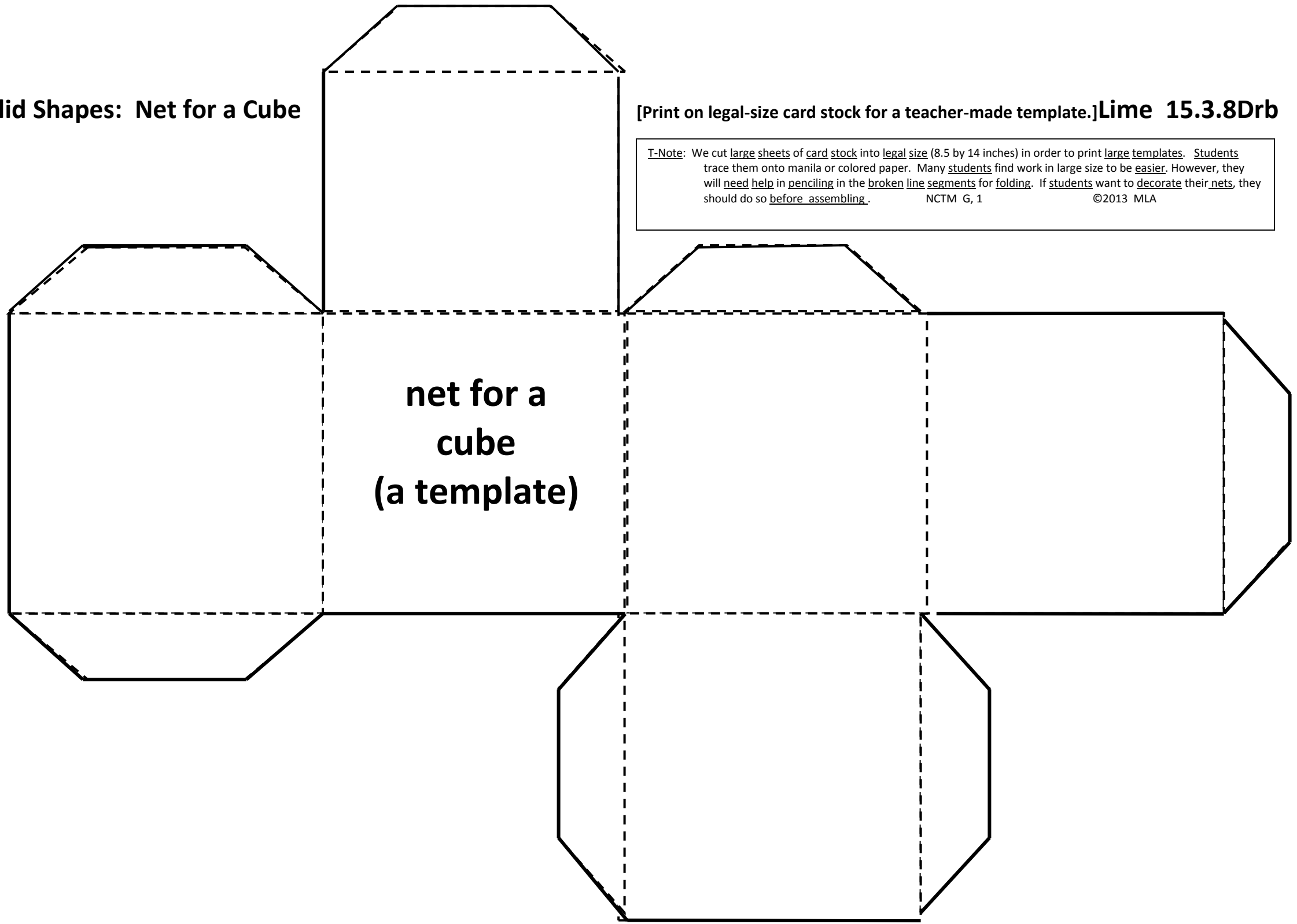
T-Note: We keep several of these nets printed onto white and colored copy paper--available in the writing-arts center for students of four years and more. They use them anytime when they have met other requirements for the day. Please also see *SM 15.3.9Dra* for how teachers can make a larger version available. ©2013 MLA



Solid Shapes: Net for a Cube

[Print on legal-size card stock for a teacher-made template.] **Lime 15.3.8Drb**

T-Note: We cut large sheets of card stock into legal size (8.5 by 14 inches) in order to print large templates. Students trace them onto manila or colored paper. Many students find work in large size to be easier. However, they will need help in penciling in the broken line segments for folding. If students want to decorate their nets, they should do so before assembling. NCTM G, 1 ©2013 MLA



Name _____

[Print on letter-size paper.]

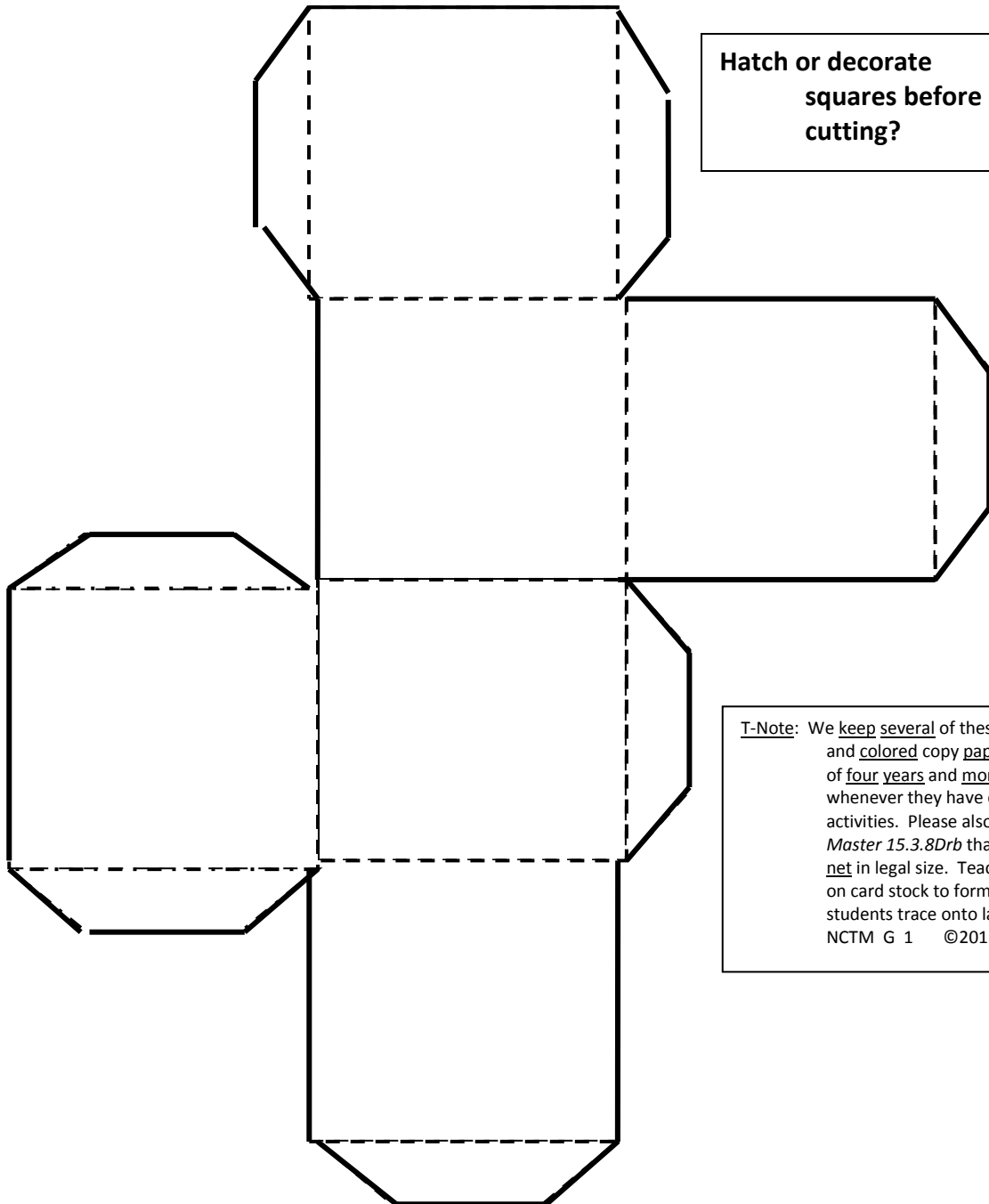
Date _____

Net for a Cube^T

Lime 15.3.8 Dra

S-Note: A net is a plane (flat) shape that can be folded into a three-dimensional solid shape.

A cube is a solid shape because it has three dimensions, a polyhedron because it has polygons for faces, a regular polyhedron because all its faces are the same, and a Platonic solid because all the vertices (or corners) are the same. Cut around the unbroken outside line segments. Fold on the inside broken line segments. Hold the squares in front of you and fold away from you. The glue tabs are trapezoids!



Hatch or decorate squares before cutting?

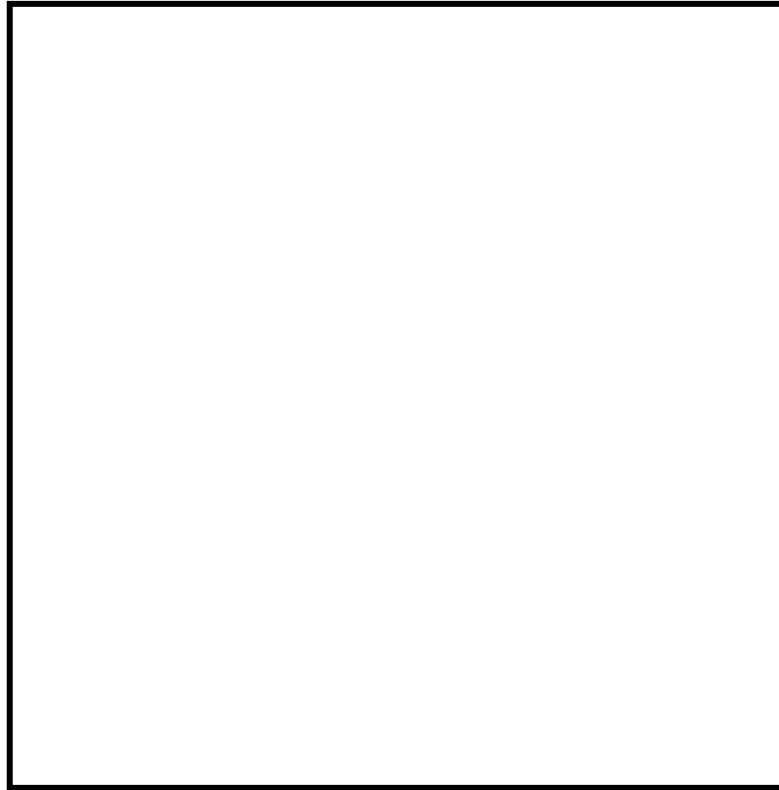
T-Note: We keep several of these printed on white and colored copy paper—for students of four years and more. They use them whenever they have completed required activities. Please also look for *Spectrum Master 15.3.8Drb* that offers the same net in legal size. Teachers print these on card stock to form templates that students trace onto larger paper.
NCTM G 1 ©2013 MLA

Name _____

Date _____

Tangram Quilt Block Square: Blank^T

Lime 15.3.7Drb



T-Note: All of the tangram puzzles that we know of to be designed for school age activities are 20 square cm or 1 dm. All of the seven tangram puzzle pieces can be fit into this dm square. However, when students design quilt blocks they need not use all seven pieces; they can use one piece or just a few repeatedly. We recommend researching geometric quilt block designs in library resources and online; many can be achieved or approximated with the tangrams as pattern pieces. Special stores for quilting and other crafts have designs in different formats as well. It may take some zooming on a copy machine to get the size of the square just right.

NCTM G1

2013 MLA

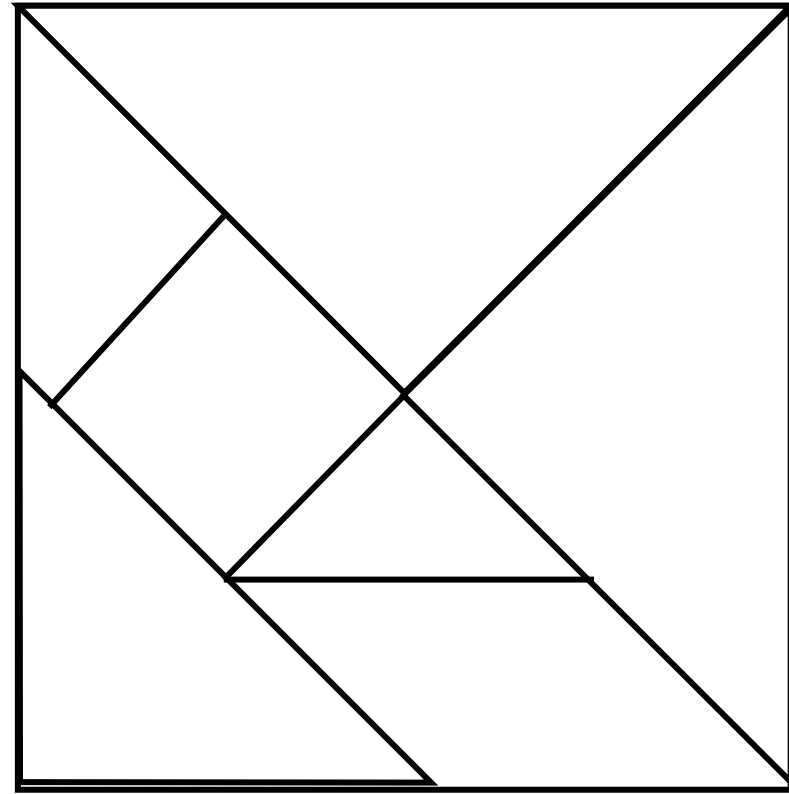
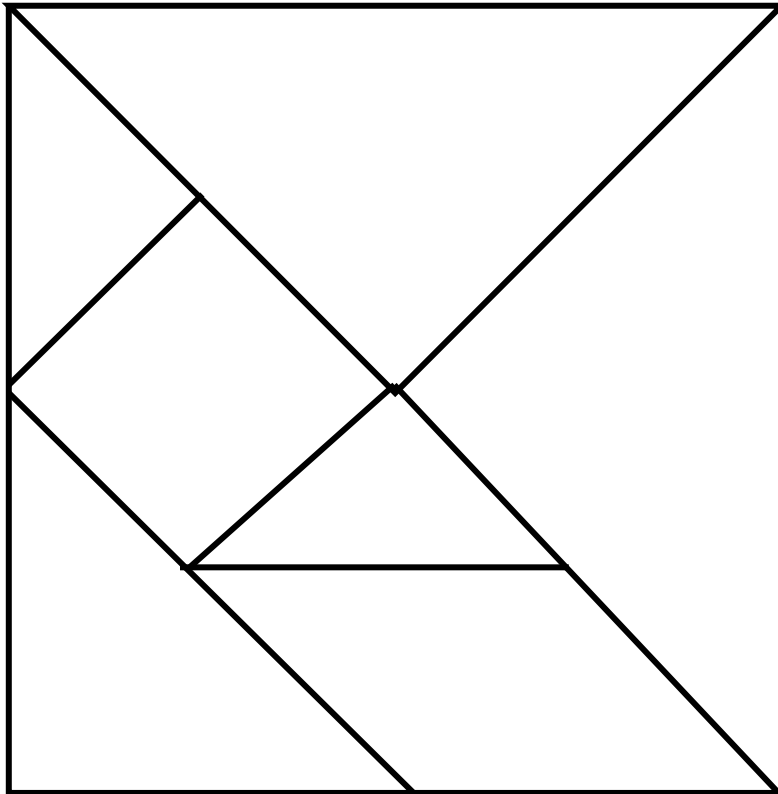
Name _____

[Will print in "landscape" orientation.]

Date _____

Tangram Puzzle:^T Filled In

Lime 15.3.7Dra



T-Note: These can be printed in several colors—those of the purchased plastic tans (red, blue, yellow, and green) or in some more exotic tints and shades. Students can mix colored tans while always checking to be sure that they have the same seven polygon shapes. When arrayed as above, the purchased plastic tans form a square of 10 cm by 10 cm that also corresponds to the dm-cm grid. ⁵www.didax.com 2-490J [4 tangram puzzles], www.schoolspecialty.com 1016623 [*Tangrams from a Jar*, design cards] NCTM G 1 ©2013 MLA

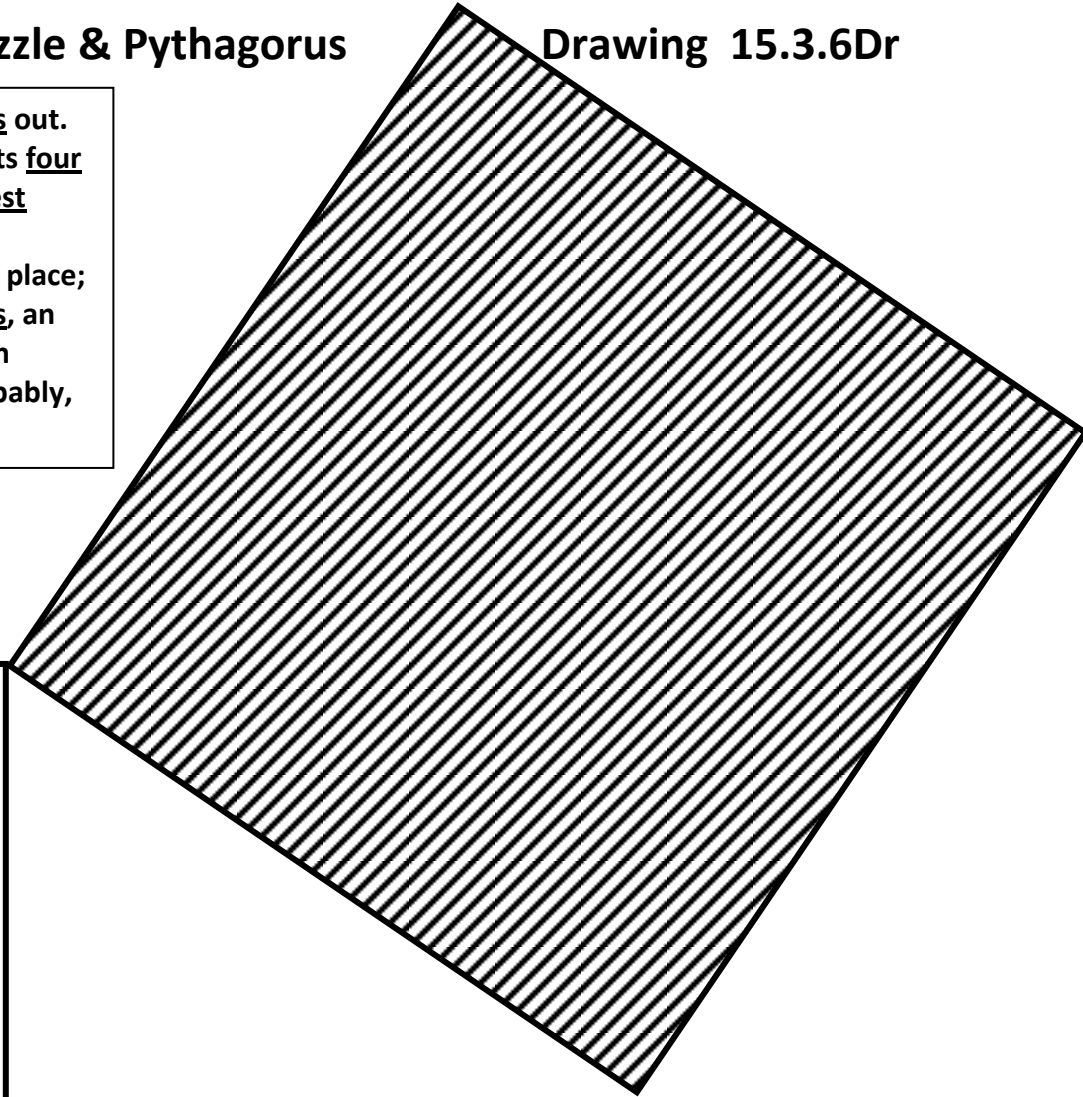
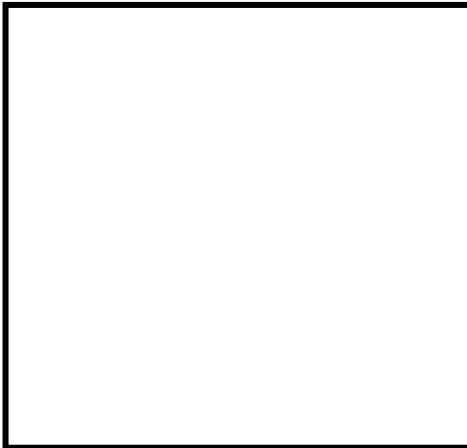
Name _____

Date _____

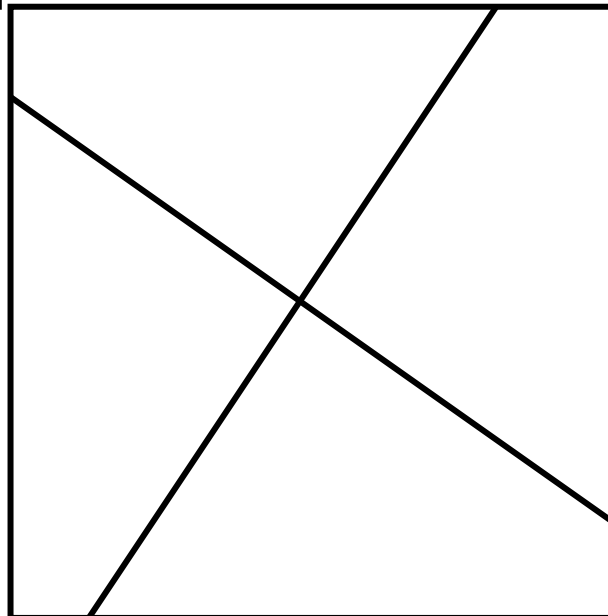
Right Triangle Puzzle & Pythagorus

Drawing 15.3.6Dr

S-Note: Cut the three squares out. Cut the medium square into its four pieces. Lay them on the largest square leaving room for the smallest square. Glue stick in place; weight overnight. Pythagorus, an ancient Greek thinker, is given credit for this puzzle but, probably, many worked on it.



T-Note: We keep several of these drawings printed on colored copy paper. Students may want to keep one SM intact as a base and glue pieces of another color onto it. Such experimentation is always welcome but should be discussed with the teacher to avoid confusion.

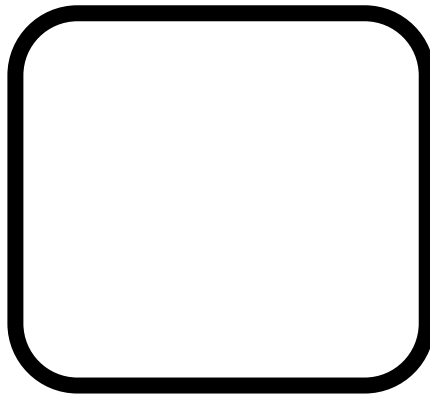
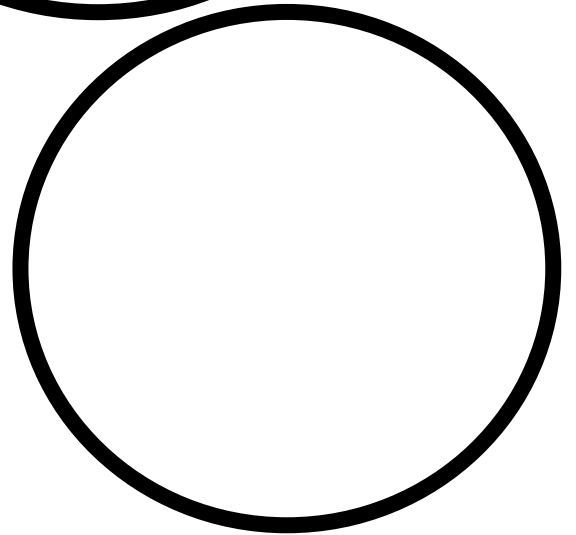
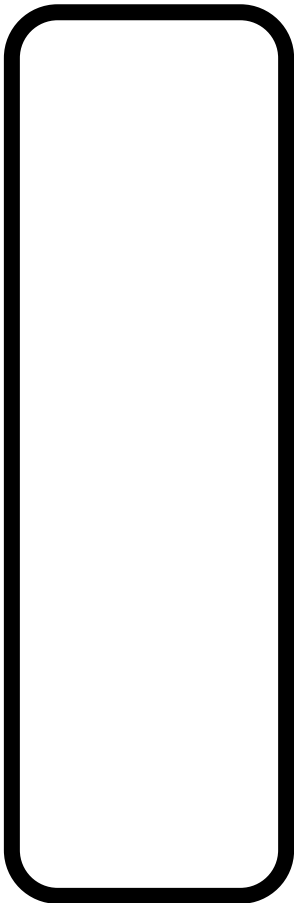
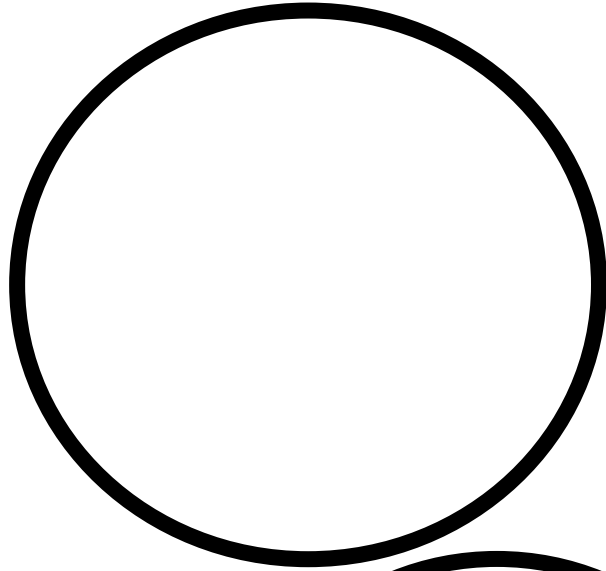
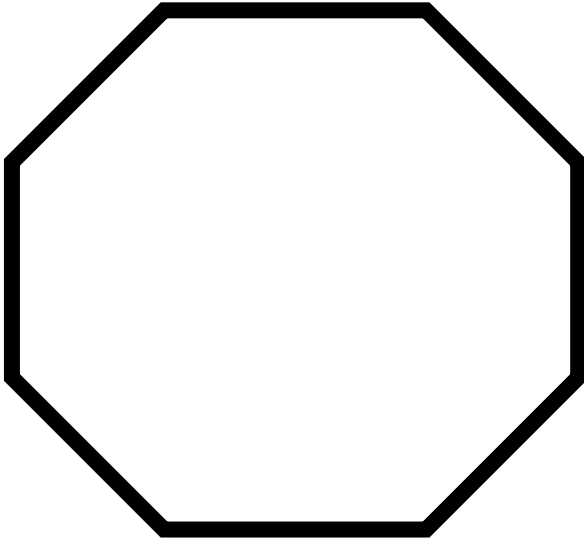


Name _____

Date _____

Street Sign Shapes

Lime 15.3.5Dr



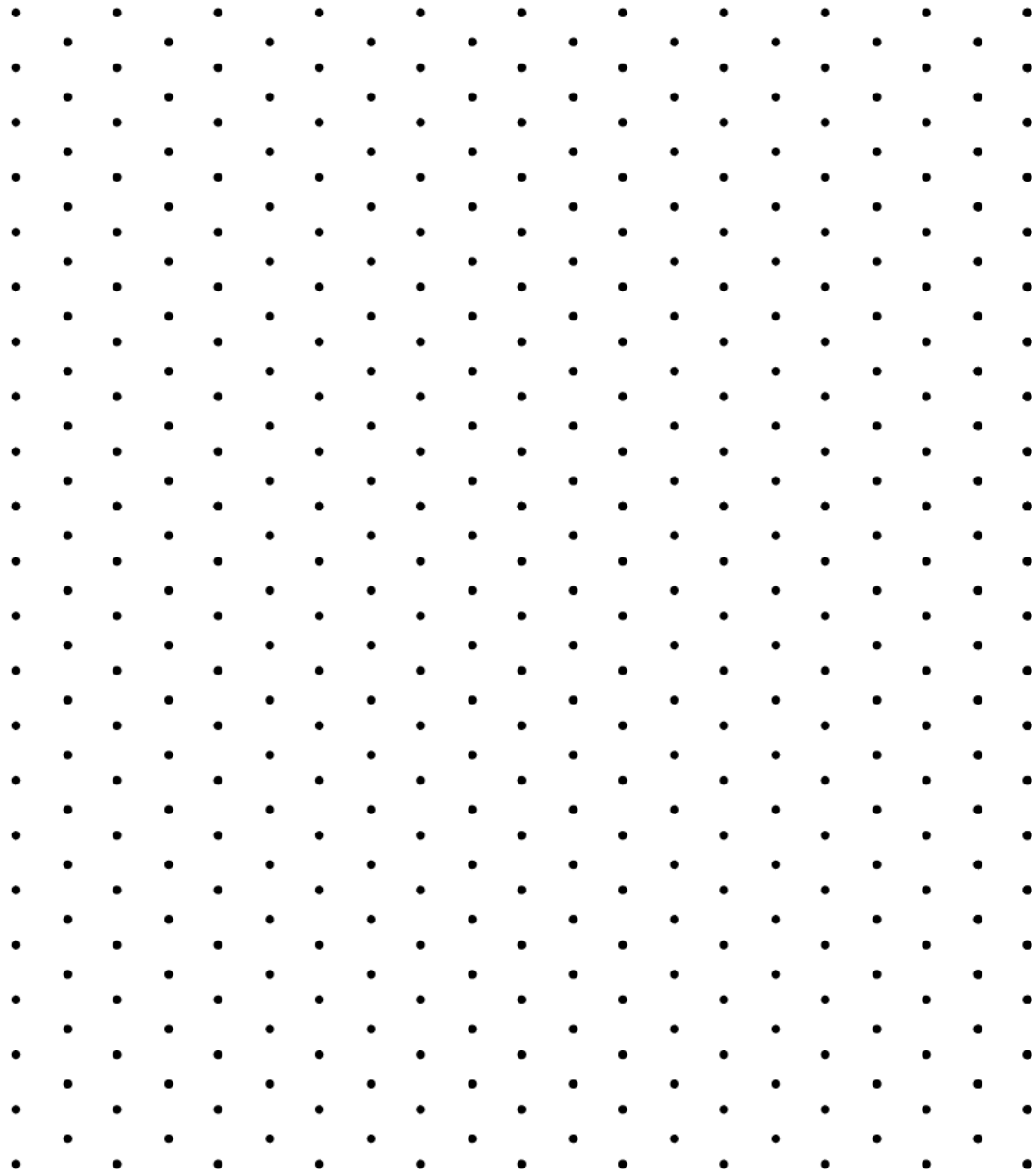
T-Note: The teacher may want to have a supply of the octagons above cut out of red paper for the stop signs. NCTM G 1 ©2013 MLA

Name _____

Date _____

Dot Matrix: For Isometric Geoboard

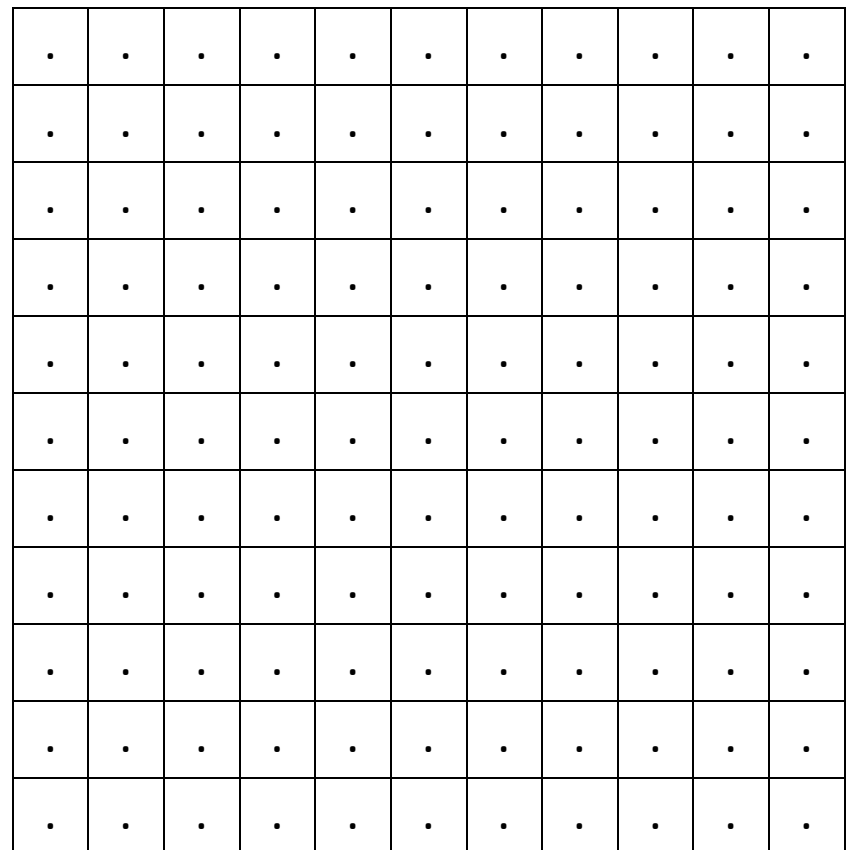
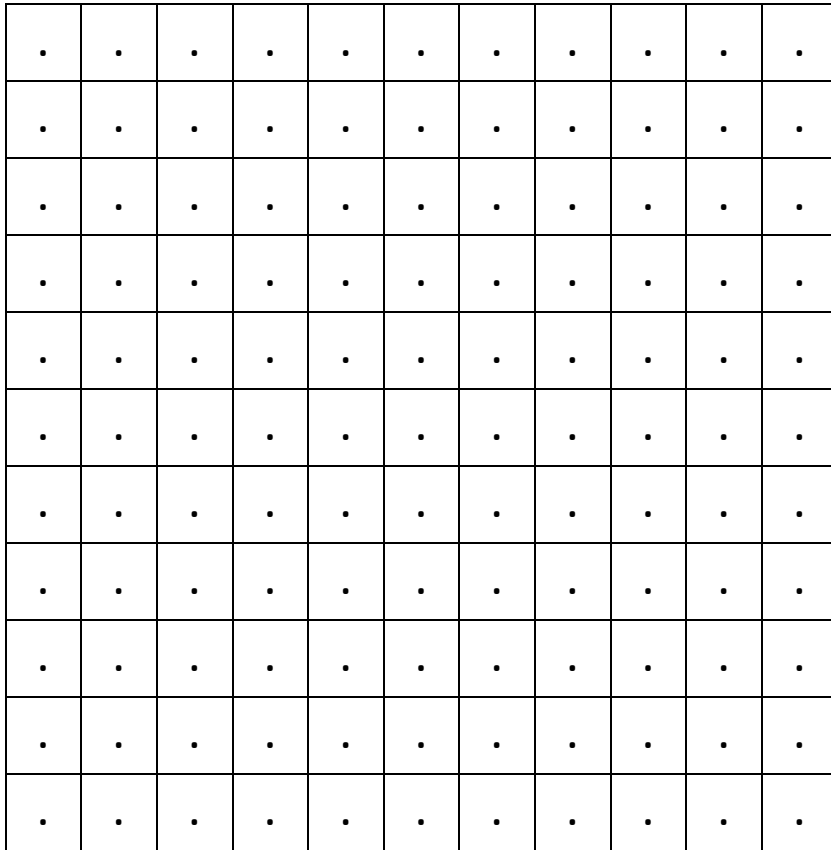
Lime 15.3.4DM



Name _____

Date _____

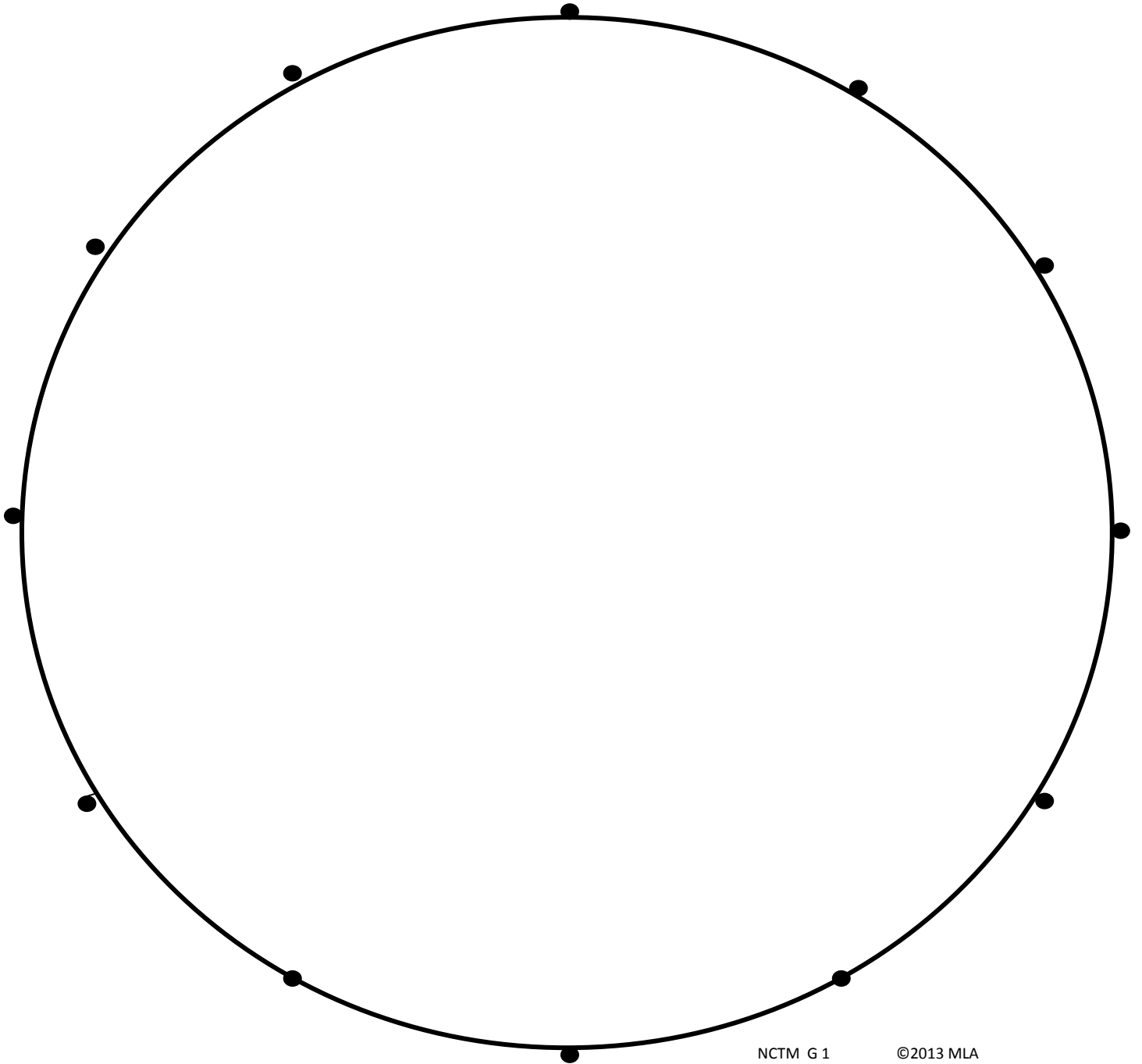
Dot Matrices:^T Square for 121-Peg Geoboard^S [letter size, landscape] Lime 15.3.3DM



Name _____

Date _____

Dot Matrix: Circular for 24-Peg Geoboard^S Lime 15.3.2DM

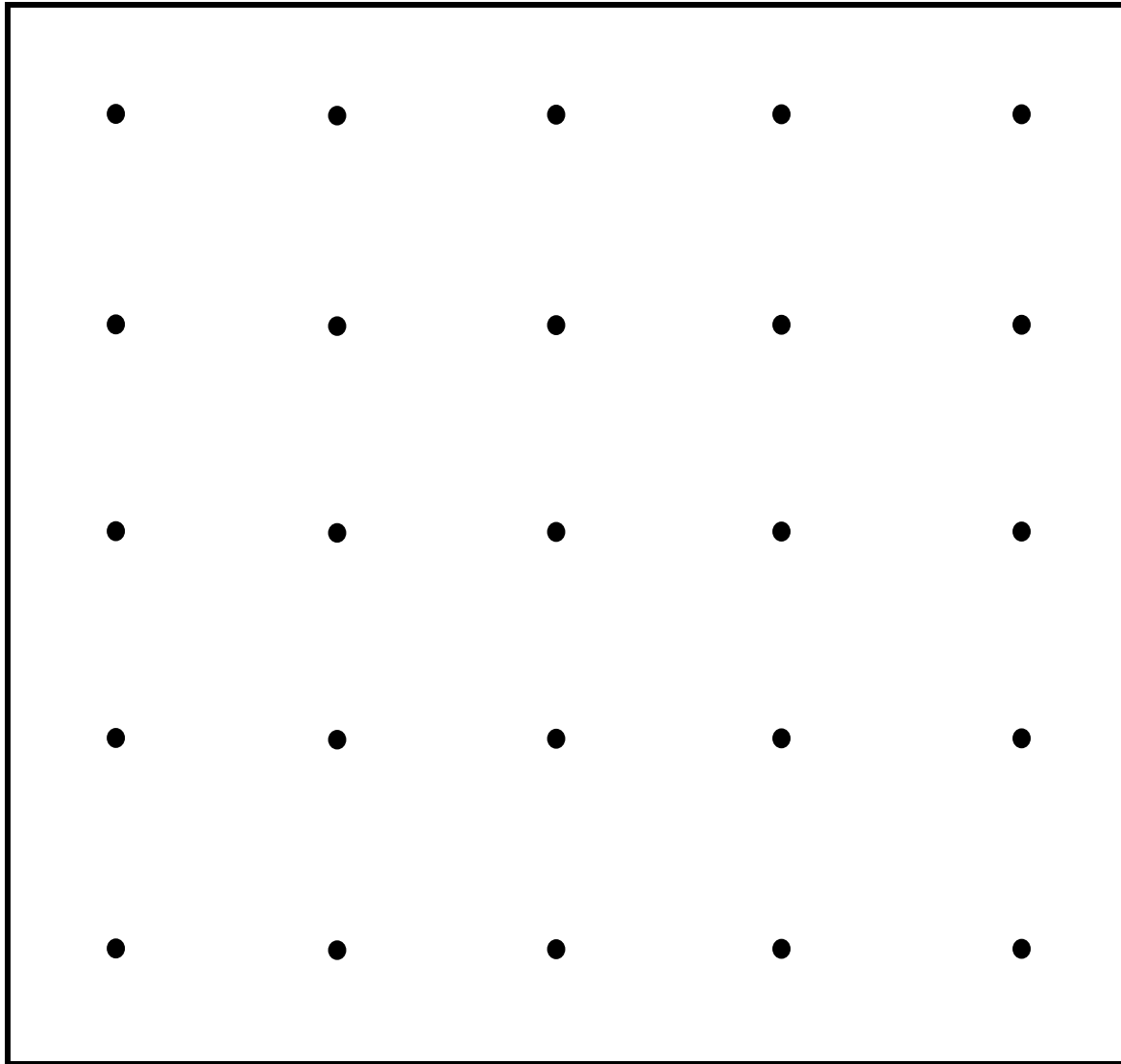


Name _____

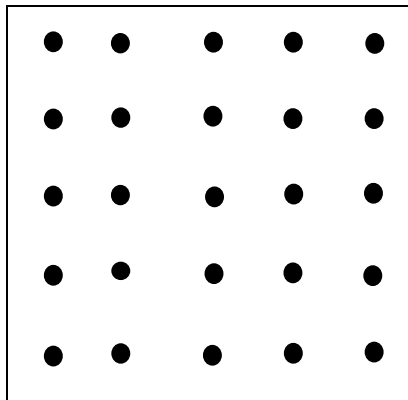
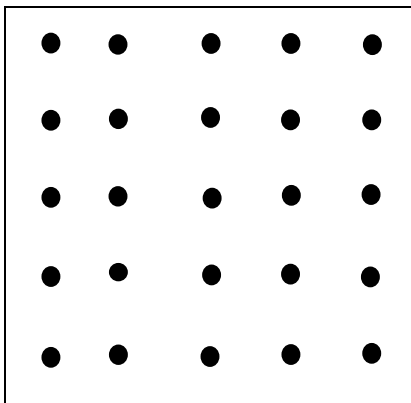
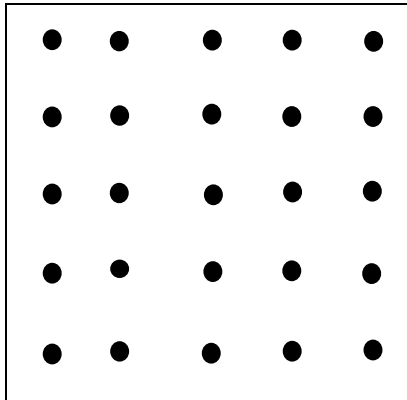
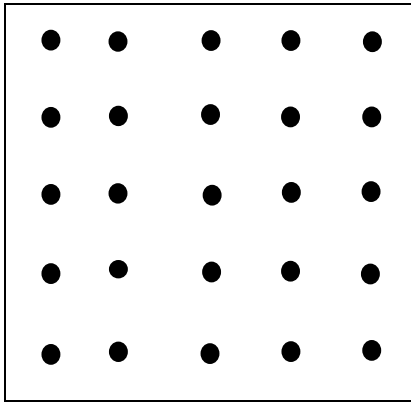
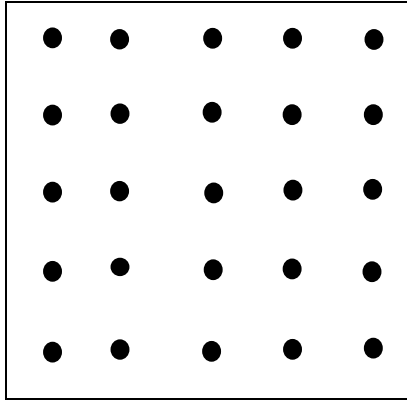
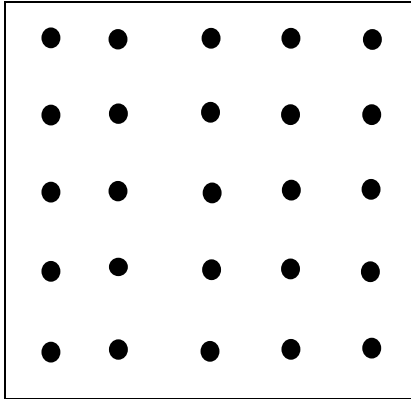
[page 1 of 2]

Date _____

Dot Matrices:^T Square for 25-Peg Geoboard^S Lime 15.3.1DM



T-Note: The larger size dot matrix may be easier for students recording the first time. The smaller size actually requires drawing to scale or “scaling down” an image. Grids can be printed on white or colored copy paper to go with the SAC Series or the colors of geoboards? ^Swww.didax.com 2-420W [square, 25-peg geoboard] NCTM G 1 ©2013 MLA



Section 5

Labels

T-Note: These are solid or three-dimensional (3D) shapes. We keep some, usually purchased sets, on display (See Didax 2-450J.). We put these labels directly onto the solids. The first four below are polyhedra, solid shapes with sides or faces that are polygons. There is an infinite set of pyramids—polyhedra with polygons for bases and three or more identical triangles for sides. **The set of solids that we use carries no choking hazard warning but some purchased or homemade sets may actually be choking hazards so we include a label to remind teachers to investigate.**

binder clips [These are used to clamp constructions from nets. See schoolspecialty.com]

choking hazards

wooden geometric solids

[DD2-450E]

cube [The cube is also known as one of the “platonic solids”—See below.]

square prism [This is an alternative name that can be added to the cube.]

rectangular prism

pyramid

cylinder

triangular prism

hemisphere

octagonal prism

hexagonal prism

cylinder

sphere

cone

T-Note: Below are labels for the platonic solids, polyhedra with identical sides. We keep some made by students, labeled, and displayed. They can be folded from nets in *SMs* or modeled from Didax Geofix pieces [210885W].

cube

tetrahedron

[This solid shape also meets requirements as a pyramid because it has a polygon for a base that is the same as the three triangular sides.]

octahedron

icosahedron

dodecahedron

Labels for Measures
[including geometric volume
shapes]

3.0.12Lb

T-Note: These measures are called for in several series and provoke interest across all ages. Teachers need to exercise careful judgment regarding hazards in measurement materials; we use many that are designed for adults so do not, customarily, carry warnings. However some may pose hazards like sharp edges, as on metal tape measures.

We keep a large covered bin of pony beads that students scoop up to measure. These are clearly marked as choking hazards for under three years, so should be available only to older children.

None of our lists of labels are designed to anticipate all possible needs but to show teachers how we organize math labs and keep them safe. Teachers are urged to check labels on new materials and label them similarly for all participants in the math lab.

pony
beads

choking
hazard

trundle
wheel

choking
hazard

clear
volume

measures

[kitchen measures]

How many ml does the
pyramid shape hold?

metric ruler & US Customary ruler [DD 197627E]

[We try to have only rulers that have both gradients on them and label several.]

meterstick & yardstick [DD 197605E]

meterstick & yardstick

meterstick & yardstick

meterstick & yardstick

[We try to have only meter sticks that also show the USC gradient. We label on both sides of each.]

non-standard units

of measurement

toothpicks, paper clips . . .

[These should stay in the teacher's tote.]

180° protractor [DD 197893B] **360° protractor**

180° protractor **360° protractor**

180° protractor **360° protractor**

[We label of few of each type of protractor.]

pan balance [DD 8-502J]

angle mirrors teacher's tote

[These are small flat mirrors designed for pattern blocks; but used in pairs in *OWaM*. We tape them together vertically and horizontally, with colored masking tape, to form angle mirrors. These angle mirrors are set up on flat 360° protractors that we also make ourselves.]

T-Note: Many sets of shapes have design cards that can be purchased to go with them. We call all such cards “design cards” to distinguish them from our own SACs.

tangrams

choking hazard

[DD 2-490], SS 1016623]

tangram design cards [SS 1016623]

color tiles

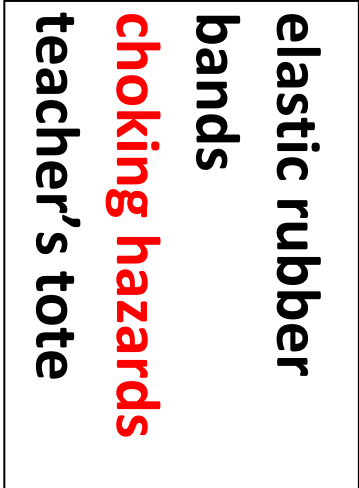
choking hazard

[DD 2-416E]

Labels for Geoboards, Pattern Blocks & Geomirrors 3.0.9Lb

T-Note: We use three types of geoboards. We label the container where they are all stored and each geoboard individually. The elastic rubber bands are clearly a choking hazard and tend to get lost. So we keep them in the teacher's tote that also contains the stapler, tape, and anything else in need of teacher care. Wherever the elastics are stored, they can have their own clear box container, labeled with the warning. We also put the warning on the geoboard container.

geoboards choking hazards



square, 25-peg geoboard
square, 25-peg geoboard
square, 25-peg geoboard
square, 25-peg geoboard
square, 25-peg geoboard
square, 25-peg geoboard

square 121-peg geoboard
square 121-peg geoboard
square 121-peg geoboard
square 121-peg geoboard
square 121-peg geoboard
square 121-peg geoboard

isometric geoboard
isometric geoboard
isometric geoboard

isometric geoboard
isometric geoboard
isometric geoboard

circular geoboard
circular geoboard
circular geoboard

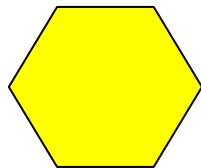
circular geoboard
circular geoboard
circular geoboard

T-Note: Here is a sign or chart for the pattern block container within an interest center—
Colors or Lime. If there are children of three years or less in the learning group, the choking hazard warning should be on the container.

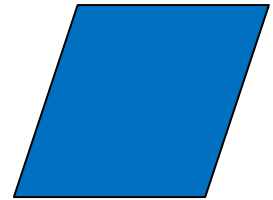
pattern blocks

choking hazards

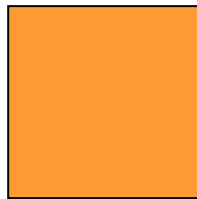
yellow hexagon



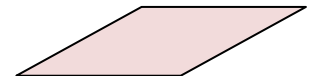
wide
blue rhombus



orange square



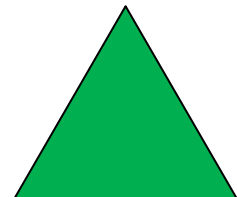
narrow
tan rhombus



red trapezoid



green triangle



design cards

[These are sets of “task cards” and “activity cards” that we call “design cards” in *OWaM* to distinguish them from our own *SACs*. They are used with the pattern blocks. DD 284530E, DD 284765E]

Geofix[®] set

choking hazard

[DD 3-205J]

geomirrors

angle mirrors

[These are stand-alone mirrors for studying reflective symmetry. DD 2-516E]

square 121-peg geoboard

**numeric
decahedral &
regular polyhedral
dice**

choking hazard_{[DD}

173341E]

Labels for Plane (Tracing) Shapes

3.0.8Lb

T-Note: We keep plane (flat) shapes in a container, sometimes with subdivisions, in the writing-arts center even though they are required in many math activities. Students use them for casual tracing activities as part of writing development and easily locate them when needed for math.

These can be teacher-made from lids of food containers—soft flat plastic that cuts with scissors. The labels below can be taped right onto plane shapes. We have also entered sketches and some descriptive material so that teachers can recognize, right away, which shapes go with which labels. The sketches can be discarded along with the **choking warnings** if the latter are not necessary for **groups of over-threes**. **We consider all of them choking hazards for under age three.**

For more on definitions and properties of plane shapes, we highly recommend the *Visual Math Dictionary* by Don Balka and others. Please see *Bibliographies*.

[The label below is for the outside of a container that holds shapes cut from lids of food containers, Montessori shapes [DD 8-1586E], purchased tracing shapes [DD 8-278J], other purchased shapes, or any combination. It also contains templates, purchased stencils with several cut-out shapes on each, called for in several activities.

The labels just below name the shapes in the Didax set of 17 shapes. There are labels for teacher-made and other purchased shapes at the end.]

plane tracing shapes & templates

choking hazard

shape tracer set French curves
(17 pieces)

choking hazard

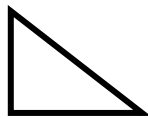
DD 8-278E

equilateral
triangle



[three equal sides]

right triangle



[one right angle of 90°]

scalene
triangle



[three sides of different lengths]

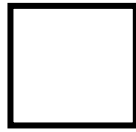
isosceles triangle



Labels for these triangles are
in smaller typeface to fit on
smaller shapes.

[two or three equal sides]

square



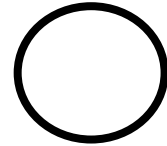
rectangle



hexagon



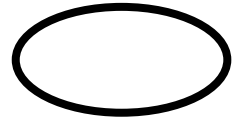
circle



parallelogram



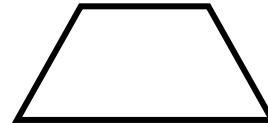
ellipse



rhombus



trapezoid

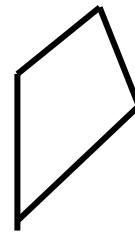


or trapezium

pentagon



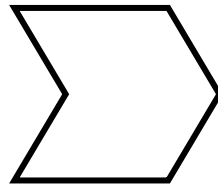
kite



heptagon [seven sides]

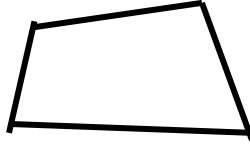
octagon [eight sides]

chevron



Teacher-made from coffee can lids and the like:

**quadrilateral
or quadrangle**



[Quadrilateral means “four-sided;” quadrangle means “four-angled” or any shape with four sides and four angles.]

acute triangle



[all angles smaller than 90°]

obtuse triangle



[one angle larger than 90°]

Other purchased shapes found at office suppliers:

large triangle

large triangle

French curve

French curve

Labels for Montessori Shapes^S

3.0.7Lb

T-Note: The “Montessori shapes” are called the “metal insets” in Montessori schools—See the website cited below for the Didax version that we use in *OWaM*. There are ten shapes in the form of (5” X 5”) plastic squares with shapes cut out of the centers, forming insets. We call their outer frames “stencils” or “templates” and the inner pieces “insets.” The outer stencils are especially helpful for writing development as their inside cut-outs guide the pencil. Students often experiment with them first and then move on to the insets. They invent all kinds of striped, colored, and textured designs to fill in their tracings—usually with colored pencils.

We put matching labels on both stencils and insets and cover the labels with clear tape. Thus we offer each label twice in the list below.

We keep the Montessori shapes in the writing-arts center with other flat shapes, used mostly in tracing activities. All can be kept in one container perhaps with subdivisions for the different sets of shapes. When students select a math activity card requiring the plane or tracing shapes, they have no trouble locating them in the writing-arts center.

Montessori shapes are labeled by the manufacturer as choking hazards for under threes.

^Swww.didax.com 8-1586E [Montessori shapes]

[The first label below is for the outside of a container that holds the Montessori shapes along with the other sets.]

plane tracing shapes & templates

choking hazard

Montessori shapes

choking hazard

circle

circle

rectangle

rectangle

pentagon

pentagon [a five-sided polygon]

square square

ovoid (egg shape)

ellipse

[curves at both ends are even]

ovoid (egg shape)

ellipse

triangle or equilateral triangle

triangle or equilateral triangle

curved triangle or curvilinear triangle

curved triangle or curvilinear triangle

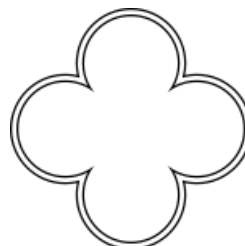
trapezoid or trapezium

[a quadrilateral with at least one pair of parallel sides]



trapezoid or trapezium

quadrafoil or quatrefoil



quadrafoil or quatrefoil

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Section 6

The City House

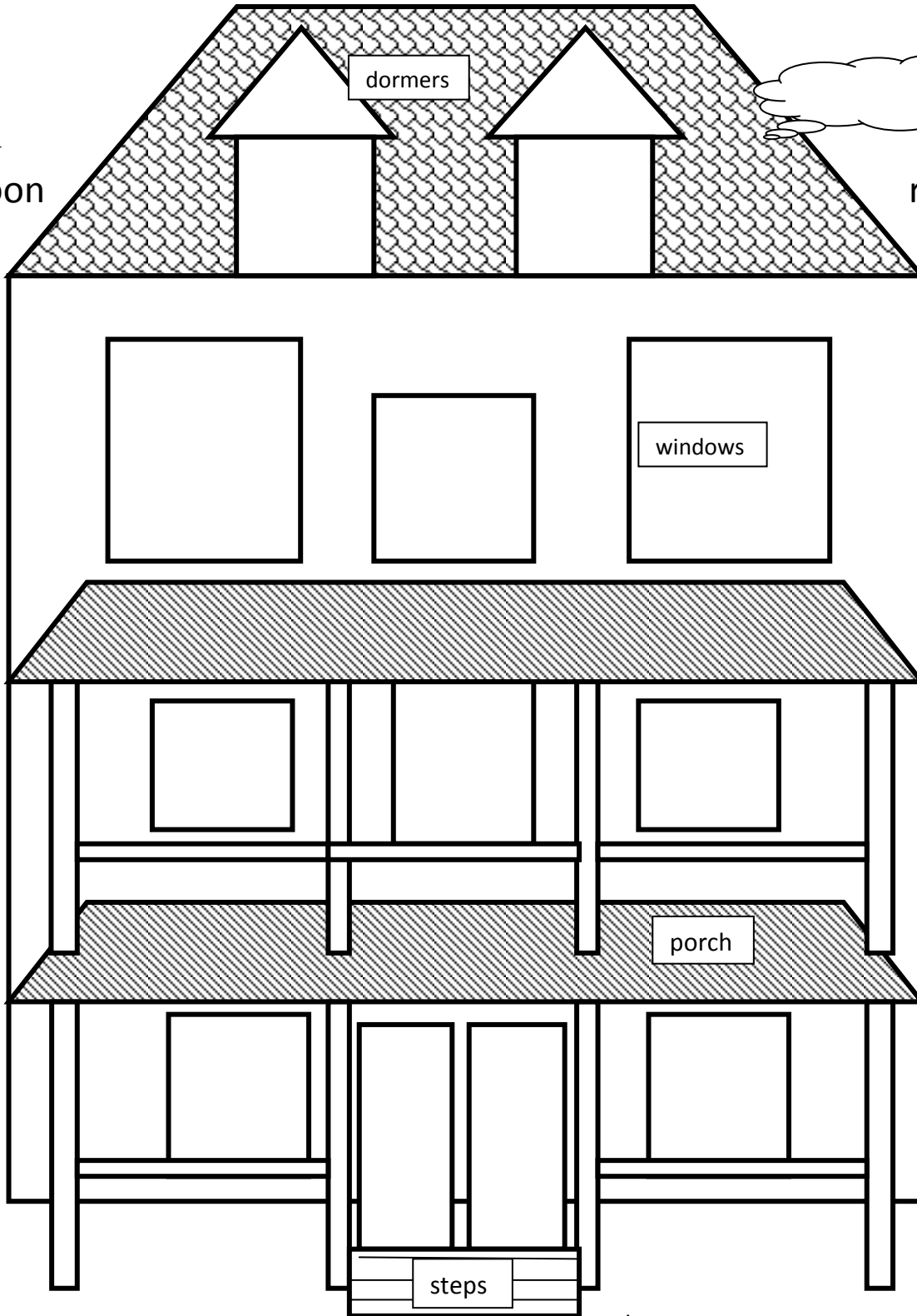
Name _____

The City House

Drawing 15.0 SM



moon



dormers

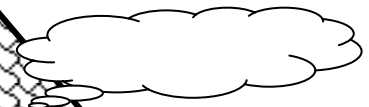
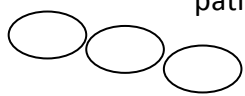
roof

windows

porch

steps

path



Name _____

The City House

Shape bank

Triangle steps

Square porch

Rectangle roof

Cloud moon

Dormers

The City House drawing is filled with shapes. Students cut out the window and doors and place construction paper behind the house and glue it. Enhance the drawing by adding extra features such as people looking out the windows, birds on the roof, or grass in the front of the house. Create a journal response.

Cubit Lengths

Lime 3.2.1FTa

The length of a cubit was based on the distance from the elbow to the fingertips, so it varied between different ancient groups of people. Here are some samples from Egypt, Babylon, and ancient Israel:

Culture	Inches (centimeters)
Hebrew (short)	17.5 (44.5)
Egyptian	17.6 (44.7)
Common (short)	18 (45.7)
Babylonian (long)	19.8 (50.3)
Hebrew (long)	20.4 (51.8)
Egyptian (long)	20.6 (52.3)

But when Noah came off the Ark, only one cubit measurement existed—the one he had used to construct the Ark. Unfortunately, the exact length of this cubit is unknown. After the nations were divided, years later at the Tower of Babel, different cultures (people groups) adopted different cubits. So it requires some logical guesswork to reconstruct the most likely length of the original cubit.

Section 7

Scope & Sequence Table

Abbreviations for Basic Materials & Budget

SAFETY & Materials for Young Children

Language Development

Curriculum Goals

An Introduction to Elementary Shapes and Measures

Scope & Sequence Table for *OWaM*

Lime 3.0.6TR

Here are represented all of the color-coded component series for students in the math lab, *Our World as Math*. The disciplines of mathematics are grouped into three broad color-coded themes:

- number and operations coded in the “warm colors” of pink, red, orange, and yellow
- geometry and measurement (including statistics and probability) coded in “cool greens”
- algebra and its branches and applications coded in “cool blues”

The youngest children participate in these themes as represented at the bottom of the table. Children of about one to four years of age may begin with *First Lights* or *Colors* that represent all three themes. At perhaps four or five years of age, the child may enjoy activities in *Pink Series Number & Operations* into the tens. Shortly thereafter, this child may enjoy *Red Series* also for number and operations but involving larger numbers. Some children participate in *Red* and *Lime Shapes & Measures for Everyday* simultaneously. Eventually they will enjoy *Cyan Pre-Algebra*. The ages identified on the table are approximate ages when children are likely to enjoy either early childhood *First Lights* or *Colors* or the beginning series in the three distinctive themes. The numerals in parentheses indicate the usual order in which students participate. These numerals also appear in the alphanumeric code that identifies each activity card in each series.

The National Council of Teachers of Mathematics (NCTM) also recommends establishing “Connections” for young people between the disciplines of mathematics and those disciplines that apply them. So, we created *Math in the Physical Sciences* coded in teal; *Math in the Social Sciences* coded in sienna; and *Math in Literature and the Arts*” coded in vermilion. Discrete math, coded magenta consists of graph and logic activities. Students can participate in all of these series along with the others in the three major themes—They span early childhood to adolescence.

Three additional series are for teachers (all parents, elders, caregivers, and educators of the young). These series are coded in the neutrals of white, gray, and black. They offer, respectively: recording formats to share with students; sources for manipulative math materials; summaries of research and bibliographies for further reading and study.

NUMBER & OPERATIONS
Yellow (6)
 number & place value to infinity, negative powers, square root, scientific notation, logarithms

Orange (4)
 number, place value, decimals, operations, roots & powers

Red (2)
 number, place value & four operations
 0 to 100s

Pink (1)
 number & four operations,
 0 to 10s

number
 as *How Many?*

number as
Sun, Moon & Stars

GEOMETRY & MEASUREMENT (9)
Green (9)
 data analysis & probability, demographics & econometrics

9 to 12 years

Lime (3)
Shapes & Measures for Everyday
 ages 4 to 9 years

Colors (0)
 shapes & measures as *Shapes & How Much?*
 ages 2 to 5 years

First Lights (00)
 shapes & measures as *Flowers, Trees & Houses*
 ages 1 to 4

ALGEBRA
Violet (10)
calculus

Indigo (8)
Math at Work
computer applications, instrumentation & programming

Blue (7)
 algebra

Cyan (5)
 pre-algebra

algebra
 as *What Changes?*

Changes as
Water, Food & Earth



Magenta (11), discrete math—non-numerical graphic & logical thinking, all ages

Teal (12), Math in the Physical Sciences, all ages

Sienna (13), Math in the Social Sciences, all ages

Vermilion (14), Math in Literature & the Arts, all ages

Scope & Sequence Table for *OWaM*

Lime 3.0.6TR

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The youngest children participate in these themes as represented at the bottom of the table. Children of about one to four years of age may begin with *First Lights* or *Colors* that represent all three themes. At perhaps four or five years of age, the child may enjoy activities in *Pink Series Number & Operations* into the tens. Shortly thereafter, this child may enjoy *Red Series* also for number and operations but involving larger numbers. Some children participate in *Red* and *Lime Shapes & Measures for Everyday* simultaneously. Eventually they will enjoy *Cyan Pre-Algebra*. The ages identified on the table are approximate ages when children are likely to enjoy either early childhood *First Lights* or *Colors* or the beginning series in the three distinctive themes. The numerals in parentheses indicate the usual order in which students participate. These numerals also appear in the alphanumeric code that identifies each activity card in each series.

The National Council of Teachers of Mathematics (NCTM) also recommends establishing “Connections” for young people between the disciplines of mathematics and those disciplines that apply them. So, we created *Math in the Physical Sciences* coded in teal; *Math in the Social Sciences* coded in sienna; and *Math in Literature and the Arts*” coded in vermilion. Discrete math, coded magenta consists of graph and logic activities. Students can participate in all of these series along with the others in the three major themes—They span early childhood to adolescence.

Three additional series are for teachers (all parents, elders, caregivers, and educators of the young). These series are coded in the neutrals of white, gray, and black. They offer, respectively: recording formats to share with students; sources for manipulative math materials; summaries of research and bibliographies for further reading and study.

NUMBER & OPERATIONS
Yellow (6)
 number & place value to infinity, negative powers, square root, scientific notation, logarithms

Orange (4)
 number, place value, decimals, operations, roots & powers

Red (2)
 number, place value & four operations
 0 to 100s

Pink (1)
 number & four operations,
 0 to 10s

number
 as *How Many?*

number as
Sun, Moon & Stars

GEOMETRY & MEASUREMENT (9)
Green (9)
 data analysis & probability, demographics & econometrics

Lime (3)
Shapes & Measures for Everyday
 ages 4 to 9 years

Colors (0)
 shapes & measures as *Shapes & How Much?*
 ages 2 to 5 years

First Lights (00)
 shapes & measures as *Flowers, Trees & Houses*
 ages 1 to 4

ALGEBRA
Violet (10)
calculus

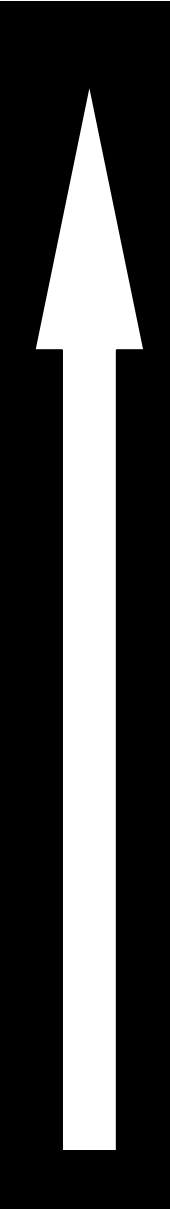
Indigo (8)
Math at Work
computer applications, instrumentation & programming

Blue (7)
 algebra

Cyan (5)
 pre-algebra

algebra
 as *What Changes?*

Changes as
Water, Food & Earth



Magenta (11), discrete math—non-numerical graphic & logical thinking, all ages

Teal (12), Math in the Physical Sciences, all ages

Sienna (13), Math in the Social Sciences, all ages

Vermilion (14), Math in Literature & the Arts, all ages

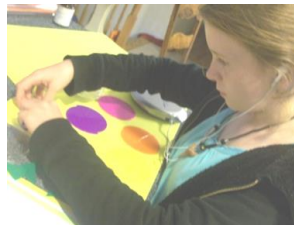
SAFETY, Materials & Young Children

Lime 3.0.4 TR

pony beads for measuring



circle fractional parts



Judy® clock



SAFETY

Please remember that nearly all of the learning materials, including art materials, associated with early childhood education are identified by the Consumer Product Safety Commission as choking hazards for children of three years and less. *Spectrum Activity Cards in Lime Series* are not annotated individually with cautions about choking hazards as are *First Lights* and *Colors Series* that are designed for very young children. These latter two series have cautions written into individual cards as part of *T-Notes*.

OUR INTENTION IS THAT THIS ESSAY SERVES AS THE CAUTION FOR THE ENTIRE LIME SERIES.

Choking Hazards & Children of Under Three Years

The Law

The Child Safety Protection Act (1979, 1994), requires purveyors of early childhood materials to label small toys and components with choking hazard warnings for children of up to three years or thirty-six months. Choking hazards are defined as pieces of less than 1.25 inches in diameter. For ball-shaped pieces, this lower limit is 1.75 inches. Balloons and marbles are cited as particular hazards. The federal agency in charge of implementing the law is the Consumer Product Safety Commission. We urge that caregivers consult this public website to stay current on any changes in the law as well as warnings for older children.

Public Input

Several state Public Interest Research Groups, who are mostly affiliated with The U.S. PIRG: The Federation of State PIRGs, have driven the toy-safety movement. We urge all parents, educators, and child care providers to contact a nearby PIRG in order to stay informed of all policy discussions concerning child safety.

Responsibilities in Parents, Educators & Child Care Providers

We recommend that teachers, child care providers, and community leaders designing and administering services for the very young, study the law and consult with the Commission before formulating their own policies on safety in learning materials. Responsible leaders may decide that a separate learning space must be designed for those of three years and younger since the warnings apply mostly to the first three years of life.

Parents caring for their own children need to understand exactly which learning materials pose hazards and write down lists of such materials that need supervision and storage out of sight.

First Aid & CPR

The Red Cross and the American Heart Association offer First Aid and CPR (cardio-pulmonary resuscitation) certification. We strongly urge that at least one adult in the learning space maintain this certification. These classes will equip this adult to rescue a child or adult who has ingested something occluding the windpipe.

The Warnings in OWaM

Our math lab materials include both items that are and are not labeled as choking hazards for under threes. Nearly all manipulatives for math carry the choking hazard warnings for up to age three (or 36 months) leaving very few as appropriate for learning programs.

For those that are labeled as hazards, we indicate so in labels for materials containers (found in *White Series*), *T-Notes* at the bottom of activity cards, *Basic Materials & Budget*. If teachers decide to use these materials, they should do so with the children, supervising carefully.

There are also comprehensive lists of materials for teachers in *Gray Series* with similar notations. As a further precaution, we insist that teachers check supplier catalogs and packaging for choking hazard warnings as they choose materials on their own.

The Contradiction for Educators & Educational Planners

Many of the learning materials long-used within child care and early education and serving child of three years and less ARE, indeed, choking hazards as defined by the law. Nearly all art supplies carry warning labels. However, our observations have been that such materials are used but under close supervision.

So, for teachers who are faced with the contradiction between the “not for under three years” requirement and the need to use materials for educational purposes, we may have some helpful suggestions.

Materials that the parent or teacher sees as necessary and appropriate for children but still presenting hazards can be organized onto a specialty marked tray to be brought to children for supervised activities and then put out of view when not in use. All arts and writing materials should reside on such a tray—the beginning of a writing-arts interest center that will evolve as children grow. “Classic toys” like dolls and big trucks may be perfectly safe but connecting cubes and some puzzle pieces are not.

Choking Hazard & Multi-Age Grouping

We caution teachers as well about materials labeled as choking hazards for under threes that may be in use by older children in the same space. For instance, some homes and child care settings span several chronological ages from one-year-olds, just walking, to middle childhood. All materials posing choking hazards to the younger children should be set aside for use in the presence of an adult.

Another way of reducing hazards may be to offer a separate early childhood learning space for children up to age three or four. While this compromises our belief in *interaging as a structural concept*, it may still allow enough interaging for natural development. This approach would also require a group and a space large enough to make the separation.

OWaM Series for older students must be reviewed carefully for hazards they may present for young children. Some series or some activities may have to be implemented in separate rooms.

Some Ideas on Choking Hazards & Young Children in the Math Lab

As a general safety policy, we recommend that children never be left alone in a learning space that contains choking hazards. If the learning group has children of three or younger, particular hazards can be displayed when in supervised use and put out of reach otherwise.

Our particular challenge in the *OWaM* lab stems from the idea of interaging--including younger and older children on cooperative learning. So this natural form of grouping requires

special vigilance on the part of the teacher. Careful organization may mean keeping many materials out of reach except when in use.

We invite young people and their teachers to consider the shapes and measures in use all around us—so familiar that children, barely old enough to walk, encounter throughout the day. Many *Lime* activities draw upon materials and devices common to home and community. However, these common items do present **choking hazards**. Art supplies including crayons carry such warnings as well when sold at retail stores.



Our own recommendation to teachers and parents is to know your children. On the rare occasions, when we have had students put non-food items in their mouths, these children have been considerably older than three years. Sorting out what is appropriate to ingest, and what is not, is a learning at least as important as those involving geometric shapes and metrics. We discuss hazards as we offer new *Series* to individuals and groups, as we gather for snacks and lunch, and at all “teachable moments,” when interest is high.

NCTM G 1, M 2, N & O 2 ©2013 MLA

Language Development Lime 3.0.3TR

OWaM Structural Concepts & Learning to Read & Write

Teachers (all grown-ups working with young learners) will notice that, in the math lab, students are learning to read and write constantly. The dialogue inspired by the math lab becomes writing in student journals and displays. These discussions fuel the reading of more activity cards, print and online research, reports and displays by other students.



Teachers and student partners encourage younger readers to read familiar words and expressions but willingly supply the spoken word to go with those that seem unfamiliar. Decoding (connecting sound with letter symbol) is supported by pointing under written word parts while enunciating the sounds they stand for.

We hope that our underlining signals to teachers and partnering students which words may need decoding or defining. Meaning can be enhanced by framing another spoken sentence with the new word in context. The reader will probably decode and understand the new word or expression successfully at the next encounter!

Even though we have striven to limit the amount of reading and vocabulary, geometry does bring with it quite a lot of specific terms. However, we have found that this vocabulary is phonetic and interesting. Students often surprise grown-ups with their scholarly language after some experience in the math lab.

Discussion

*Discussion is a constant in the OWaM lab. The environment is so richly structured with interesting and well-organized materials as to focus discussion on substantive content. There is a steady hum of voices of students and their grown-ups focused on materials and activities. So there is little reason to quiet individuals or groups. T-Notes contain ideas for *sparking discussion* in addition to details about materials. The carefully structured environment and activity discourage extraneous topics like gossip.*

Partnering & Discussion

In order to encourage *discussion*, each set of activity card directions (really more like suggestions) is addressed to “two friends.” This is not to restrict collaboration to only two students at a time but to encourage, at least, two young people to share activities. Many students of less than ten years can successfully collaborate in teams of three or more.

The benefits of such collaboration are abundant. Different students bring different early experiences to bear on the project; some have learned sophisticated vocabulary from other sources that can be shared; many will form insights quickly and share. Discussion fixes ideas in memory. Mathematicians and scientists work in teams!

Vocabulary for Geometry (Shapes) & Measurement

There is a great deal of geometric vocabulary in *Lime Series*. We believe that we have defined mathematical terms in the cards. Definitions are presented mostly in “text boxes,” a *Microsoft Word* facility that allows for framing terms, definitions, and concepts. There may also be arrows pointing to pictures that help define the words. Teachers can review the SACs, including the text boxes, before offering the card series or clusters of related SACs to students. If more detailed definitions are required, these can be pursued online, in college dictionaries, and math dictionaries—See *Bibliographies*. For day-to-day use with students, we find the *Visual Math Dictionary* by Don Balka and others most helpful as concepts are clearly illustrated.

Even though students’ math vocabularies will expand enormously in *Lime Series*, we do not “hold them responsible” for terms at any particular time. Clusters of cards offer repeated terms that will gradually become second nature to children. They will recall much of it as they move on to additional series within *OWaM* and many secondary and college courses.

Writing, Recording & Displaying

Throughout *Our World as Math*, students are invited to display their tape recorded investigations as well as make entries into their journals. There are advantages to both. Students who have just completed an activity and are excited about having just learned something become teachers as they form displays for a wall or bulletin board in the learning space. Those who choose to write and draw, on a small scale, in their journals may want to take their journals home to review with parents. This latter kind of *sharing* keeps parents informed and affords a natural opportunity for students to review and fix their own ideas. Some students simply prefer to record in their journals over time and review the whole with parents when they feel ready.

We keep several purchased sets and teacher-made plane shapes in a large labeled container in the writing-arts center as they develop writing. Students have little trouble locating them for math activities as well.

In tracing plane shapes and drawing with compass and straightedge, we urge students to fill their drawing papers with shapes and patterns. They learn of many new relationships and other possibilities this way as well as the discipline of getting the most out of materials and experiences.

Punctuation

Nearly all punctuation found in general use is also found in the *Lime Series*. The latest authority on grammar and punctuation for standard writing in journalism and most forms of non-fiction is *A Manual for Writers of Research Papers, Theses, and Dissertations* (7th edition) deriving from standards set by Kate I. Turabian at the University of Chicago. We use this source because we believe it most useful for students in research and writing.

Hyphens. **Modifiers** (adjectives) with **number words** coming **before** the words they **modify**, are **hyphenated**, for instance, a “two-student display.” We hyphenate two-word **fractions** like “three-fourths” but “half” and “quarter” are only hyphenated as modifiers. Teachers can suggest entering the hyphens in children’s numeric expressions where appropriate.

The **colon** appears in materials lists for students.

Commas separate items in the materials lists and elsewhere, taking the place of “and.”

The **semicolon** also takes the place of “and” in series of ideas.

The **dash** (--) has proven useful for connecting ideas that are too closely related to separate into sentences—They often show different aspects of the same idea.

Parentheses are used, within sentences, to enclose words that are synonyms for preceding words but **not** to signify multiplication, as in algebra.

We make liberal use of **question marks** to signal tentative ideas—invite a questioning approach to what otherwise might be taken for granted.

Apostrophes mark possession as in “students’ art work.” They also mark where letters are left out in **contractions** like “doesn’t.”

Quotation marks set off one- and two-character symbols like numerals that otherwise might not stand out in a sentence. They also set off words that are meant to be considered as linguistic elements like “doesn’t” in the example above.

Exclamation points are used to express surprise. They, like question marks and contractions, make SAC language resemble children’s familiar spoken language. But we’re careful not to overuse the symbol as it also represents the *factorial* in advanced number and operations (6! is read “six factorial” and means $6 * 5 * 4 * 3 * 2 * 1$).

Italics in *OWaM* indicate parts of our own copyrighted work and unique ideas. They also identify expressions as coming from other languages like *Système Internationale d’Unités* to describe the metric (SI) system of measures.

Ancient Root Words & Prefixes

Mathematicians have always drawn words, as well as ideas, from the ancient Greeks and Romans. The Romans adopted many Greek words and ideas into their Latin language. The Romans built a vast empire that incorporated all of the Mediterranean world including

Greece and, eventually, stretched as far as England. Many of the Greek words came into English through Latin and the Roman Empire. For instance “polygon,” that means “many-angled,” came to English from Late Latin and, into Latin, from Greek. We encourage the Latin plurals like “rhombi” instead of “rhombuses” because we believe knowledge of the ancient forms to be a sign of scholarship. The plural of “vertex” (for one corner of a shape) is “vertices” (for more than one). Many mathematical terms and the concepts they represent came from, as far back as scholars know, Sanskrit, the ancient language of the Indian subcontinent.



Here is one type of compass that is not marked as a **choking hazard**. It also has no sharp points.

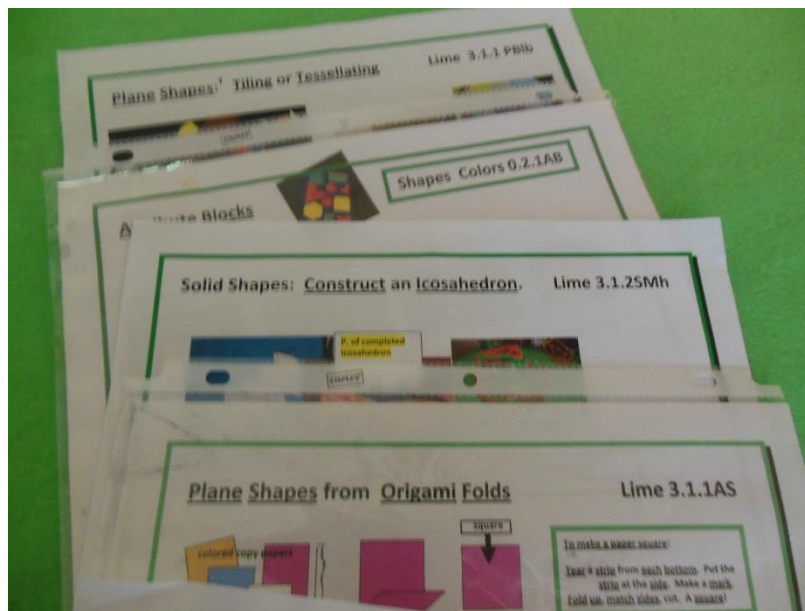
Some expressions are typical of mathematicians. For instance, “to swing a circle” or “swing an arc” means to use a compass and pencil to form a circle or curve. Such expressions are defined, used in context, and, often, further explained in *T-Notes*.

Special Uses of Words

Mathematicians use “volume” and “capacity” as separate words for separate concepts. Specifically, volume is the space taken up by a solid shape but capacity is the amount of liquid or other material that can be contained inside, if it is hollow. We have never known elementary age students to grasp that difference. So, we use the term “volume” to describe solid shapes—“This cube is 25 cubic cm in volume.” We also use it to describe liquid measures. For instance, we say, “A liter of water a measure of volume.”

In some activities, we make use of activity cards composed by other authors. These are sets of cards with graphics showing ways to arrange geometric components like color cubes. We call such cards “design cards” to distinguish them from our own *Spectrum Activity Cards*.

Abbreviations



OWaM incorporates two types of abbreviations. Apparent first to teachers and learners are our own invented abbreviations for materials. For instance, “PBI” means “pattern blocks,” and “TS” means “tracing shapes.” These abbreviations appear as capital letters in alphanumeric codes in the upper right corners of *SACs*. They are also listed with their meanings in *Basic Materials & Budget*, a *TR* essay known as a *Teacher Essay*.

We use “CC” as an abbreviation for “connecting cubes.” These are cubes with connectors for forming long sticks of cubes. For some, this made-up abbreviation may cause some confusion as it can also mean “cubic centimeters” as used in hospitals and elsewhere for measuring gases like oxygen.

The second type of abbreviations are conventional and used the world over. For instance, the international metric abbreviations like “m” for “meter,” a measure of length, and “l” for “liter” (sometimes “L”), a measure of volume, are listed in tables in college dictionaries and on the Internet. We have striven NOT to use conventional abbreviations for US customary measures because we found adding yet another set of abbreviations caused confusion among *OWaM* participants.

US customary measures for volume, weight, length and distance were formerly known as “English” but are no longer used in the United Kingdom. They are used in the North American kitchen, stores, and on highway signs.

Teachers may find it helpful to purchase or create a poster with both metric and US Customary terms, their abbreviations, and equivalencies. We keep such a poster on the wall of the learning space for all ages and participants.

Curriculum Goals: Shapes & Measures for Everyday

Lime 3.0.1TR

Shapes & Measures for Everyday is part of a comprehensive math lab called *Our World as Math (OWaM)* that offers fourteen different sets of activities represented on cards, color coded for several math disciplines and age spans of students. *Shapes & Measures for Everyday* is color coded as “Lime” in the *OWaM* math lab.



Shapes and measures are often first learned in the kitchen. Indeed, *Lime Series* shared activities, dialogue, and recording seem very much at home there. But an interest center for concentrated learning can be organized in any space indoors or outdoors.

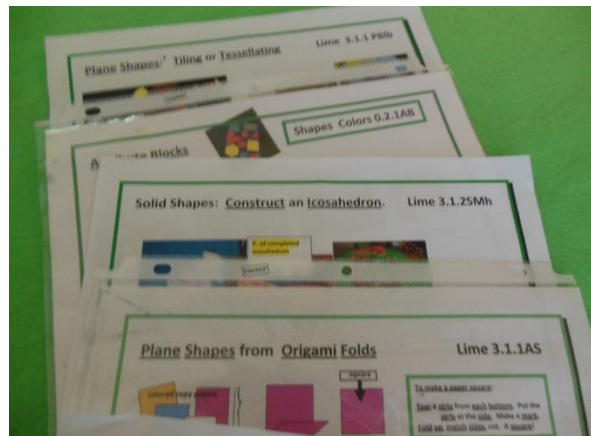


This interest center has been organized in a learning space in a home. A table and two carts offer, from left to right, materials for plane shapes, solid shapes, and measures. Students and teachers find materials there after choosing activity cards from *Shapes & Measures for Everyday*.

A writing-arts center can be on another cart or table nearby. This center holds all that is necessary to record findings from the math activities.

Teachers--Getting Oriented to the *Activity Cards*

Spectrum Activity Cards, Alphanumeric Codes & Microsoft Document Lists



Shapes & Measures for Everyday, like all series within *OWaM*, is organized by a system of activity cards composed as *Microsoft Word Documents*. *OWaM* activity cards are called *Spectrum Activity Cards (SACs)*. Each card has a distinctive alphanumeric code in the upper right corner.

SACs are grouped into *Microsoft Word Folders*. The alphanumeric codes appear as the first portions of *SAC* titles in the lists of *Microsoft Documents* for *Microsoft Folders*. So the *Folder* list also functions as a table of contents of each series.

Generally, there are four parts to the alphanumeric code:

The first numeral codes the title of a series (“3” is for *Lime Series, Shapes & Measures for Everyday*).

The second numeral identifies a cluster of related concepts within the series.

The third numeral distinguishes among particular concepts as developed on particular *SACs*.

Lastly, the capital letters are our own abbreviations for the core materials. These abbreviations are decoded in the *Teacher Resource (TR)* essay called *Abbreviations for Basic Materials & Budget (3.0.5TR)*.

Some cards have lower case letters at the far right that distinguish similar cards and materials from one another.

Specifically, in *Shapes & Measures for Everyday*:

The “3” that comes first in the alphanumeric code in all the cards simply means “*Shapes & Measures for Everyday*” or “*Lime Series*,” as opposed to the many other *Series* in *OWaM*.

The second numeral identifies the clusters of concepts. “1” is for shapes, “2” is for measures, “3” is for fractions, and so on. A “0” in this place means the same as *TR* or *Teacher Resource*

The third numeral gives subdivisions of clusters—For instance, “1” in the third place means “plane shapes” as opposed to “2,” meaning “solid shapes.”

The capital letter portion of the code identifies the principal materials required—our own made-up abbreviations of, for example, “GB” for geoboard, “FM” for found materials, “PBI” for pattern blocks, and so on—See *3.0.5TR* for a complete listing.

Some codes end with one lower case letter (a, b, c . . .) to further distinguish among similar activities involving the same materials.

Students and teachers can plan cooperatively for weeks and months at a time by means of the *Student-Teacher Cooperative Plan*.

T-Notes: At the bottom of each card teachers will find details about purposes for the activity, sources for materials, NCTM standards, and copyright.

^Swww.didax.com 210927W (geoboard) ©2013 MLA

Shapes & Measures for Everyday Folder List & Table of Contents:

3.0.0 At the top of the list are titles for several *Teacher Resource* essays, like this one, to help teachers get started. They are identified, in the *Folder* list, with the numeral “3” for *Shapes & Measures*, then a “0” meaning for teachers, and a third numeral to distinguish each essay from the others. “TR,” at the end, stands for “Teacher Resource” and is an additional indicator that the essays are for teachers rather than students.



There are also “TR” Documents that are sets of labels for the several sets of geometric materials that have many pieces. These labels appear in the list of Microsoft Documents with the same “3” for the series, “0” for teacher material, and a third numeral that distinguishes labels for one set of from another. The letters, “Lb” stand for “label.” We print them on white card stock. We include them with other necessities for teachers getting started as organization matters in a math lab and labels teach vocabulary.

Just below is the list of *SAC Documents* for students grouped by concept clusters:

- 3.1.0 Shapes (plane and solid geometry)
- 3.2.0 Measures (length, area, angles, weight, volume & capacity)
- 3.3.0 Fractions (used with US Customary or “English” measures)
- 3.7.0 Projects (applications of ideas learned from more than one group above)

Curricular Overview for Lime Series

Shapes & Measures for Everyday gives young learners and their grown-ups a systematic way of sharing, discussing, and functioning within the world immediately around them—a world that is shaped, constructed, and measured. Participants select activities that will help them frame and answer their own questions. Many such questions may be forming in the minds of children who have already worked simple puzzles, built with blocks, and investigated

measures, pots, and pans in the kitchen. We strive to connect activities on cards with such questioning by making purposes clear in titles and pictures; these should invite participation and, then, carry along the reading. Participating in *Lime* activities means investigating shape and measures in the home, school, and community.

Young people will build upon *shared activity and dialogue* toward the abstractions normally associated with higher learning.

In *Shapes & Measures for Everyday*:

We relate names, purpose, and meaning to common shapes, their measures, and part-to-whole relations.

We provide experiences with fractions as they are indispensable in everyday measurement.

We emphasize the importance of signs and symbols for caution and safety in the home and community.

After experience with the activities, students generally can:

identify plane (flat or two-dimensional) and solid (three-dimensional) shapes in relation to furniture, buildings, community;

help with measures and their fractions in the kitchen;

notice cautionary signs in the community and labels on products.

In sum, the goal is to give young learners a reading on their immediate environment, to encourage *cooperative self-governance* within that environment and to demonstrate how to stay alert to what's going on around them.

Students Doing & Learning in *Shapes & Measures*

3.1 Shapes

Geometric shapes are simply those that can be described with geometric vocabulary—like “straight,” “curved,” “angled,” “square,” and so on.

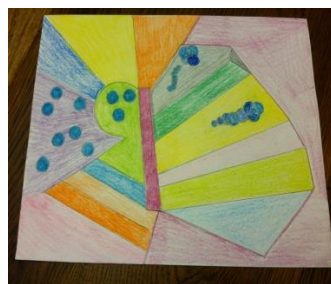
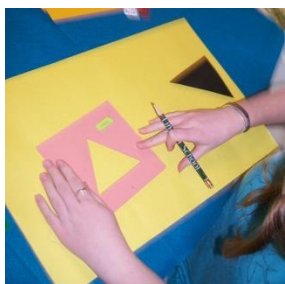


Students learn that these geometric shapes are formed of points and lines that are really invisible—but we can represent them in drawings with pencil marks. The “lines” that we draw (and see around us) are really line segments—parts of invisible lines that extend infinitely in two directions. If students appear “not to get it” when we discuss this invisibility of points, lines, and plane shapes (described below), we explain it from time to time but express no disappointment when they seem confused. They will eventually understand.

Three or more straight line segments can form closed shapes called polygons. Polygons are shapes with straight sides and two-dimensions, length and width. These polygons, too, are invisible if one were to view them from one of their sides—They have no thickness or third dimension. These shapes can be described as plane—so flat that they are really only abstractions in our minds. If any of the sides are curved (not straight) the shape may be plane but not a polygon.

Materials & Activities for Plane Shapes. The flat shapes used in the math lab are both purchased and made of recycled, soft, sheet plastic (lids of cans). These shapes are “flat” in the sense that they have little thickness but clearly have the other two dimensions of width and length. If we say to children that they actually have a third dimension of thickness even though it is very small, they don’t always understand. So we call such shapes “flat” rather than the geometrical “plane.”

A plastic container can be used to store various kinds of flat shapes tracing activities. We keep several purchased sets and teacher-made sets of them in a large labeled container



in the writing-arts center because these materials are useful in developing writing. The container is actually labeled “tracing shapes” as they are flat, used for tracing, but not actually plane. Students are free to use them anytime and have little trouble locating them for math activities as well.

In tracing flat shapes and drawing with compass and straightedge, we urge students to fill their drawing papers with shape and pattern. They learn of many new relationships and other possibilities this way as well as the discipline of getting the most out of materials and experiences.

Students also investigate the characteristics of polyhedra (the singular is polyhedron.). These are, clearly, three-dimensional shapes with polygons for faces. Some examples are cubes and pyramids. Most standard block sets include polyhedral—cubes and rectangular prisms (brick shapes). Regular polyhedra have the same faces all around. Platonic solids have the same faces all around—These faces have the same sides and angles within them.



Materials & Activities for Solid Shapes. Blocks are among the classic toys of childhood. Purchased sets nearly always include cubes and other polyhedra. We value a large set of wooden blocks as part of the math lab, perhaps, situated between math interest centers and the writing-arts center as they relate to both. The youngest students are attracted to the blocks area, usually, on arrival in the math lab. For them, construction may be a central part of their intellectual day; for older students, block construction may be a relaxing activity after more abstract investigations. All are encouraged to discuss their work and represent it in writing and art work to be displayed or entered into journals.

In many activities students construct solid three-dimensional shapes by cutting and folding paper. This paper folding is guided by two kinds of black line masters that we call *Spectrum Masters*. They appear at the end of the list of *Documents* in *Lime Series*. The first kind is simply printed by the teacher on letter-size paper of white or different colors; we keep several copies always available in the *Lime* interest center. The other kind is, most easily, printed once by the teacher onto legal-size paper and then turned into a card stock template to be traced onto art paper by students onto large drawing paper. In both types the outline of the faces of the solid shape is called a net.

3.2 Measures



The NCTM (National Council of Teachers of Mathematics) points to two aspects of measurement:

One is that materials and objects can be measured for different attributes. For instance, a book can be weighed or measured for length (width, and depth). Such distinctions are abstract notions for most preadolescent children. So teachers may decide to place a poster near the large blocks and in other relevant areas posing questions like: Weigh the blocks or measure with a ruler? Deeper understanding of attributes and measurement are developed in *Green Series*.

The second point made by NCTM is the need for precision in measurement. Just as the understanding of attributes requires abstract reasoning, the understanding of precision may require keen perception and patience found in greater abundance in adolescents than in young children. However, teachers, in the *discussion* phase of any *Lime* activity, can encourage students to check on each other's measures to confirm how precise an initial measurement has been.

In general, the importance of selecting meaningful attributes to measure and precision in measurement will come naturally over time with vast experience offered by the math lab.

Another important way of describing measurement is that it is really a matter of iterating or repeating smaller units in measuring larger ones. Further, the choice of the units is really governed by those who measure. Some are standard or agreed upon within communities and over time like inches and centimeters. Others can be determined by circumstances like "pacing off" a parcel of land. Such measures are called non-standard measures.

Students measure area in tiling activities by iterating (covering) surfaces with plastic squares of uniform size. They also learn to multiply length times height to calculate area.

A great deal of attention is given to US customary kitchen measures (and their fractions). We believe this gives meaning to kitchen *dialogue* for young children. *Shared* and **safe** cooking activities are an important growth experience in childhood. A kind of proxy for cooking (that often involves high heat and sharp utensils) is the pouring and measuring of "pony beads" that are acquired at crafts stores—These, however, present **choking hazards** that must be taken into account as we offer them in the math lab.

Comparing USC and metrics across several experiences makes *connections* by learners that last. Students form what NCTM calls “benchmarks,” mental images of, quarts and liters, ounces and grams—and, in the auto, miles and kilometers.

For measurement of angles, we offer students a variety of activities that feel like arts and crafts activities. They draw and measure angles with protractors. They draw circles with safety compasses. Advanced concepts in angular construction are part of another *OWaM Series* called *Indigo Math at Work*.

We offer both “volume” and “capacity” as vocabulary for measurement. Volume appears in the context of construction when one needs to reason the outside shape of a three-dimensional solid shape. Capacity refers to the inside space of a vessel, usually a device to measure liquid or pourable ingredients in cooking.

3.3 Fractions

In *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics states that fractions can represent four ideas in number “as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers.” (NCTM, 2000, Web Access Feb.27, 2014)

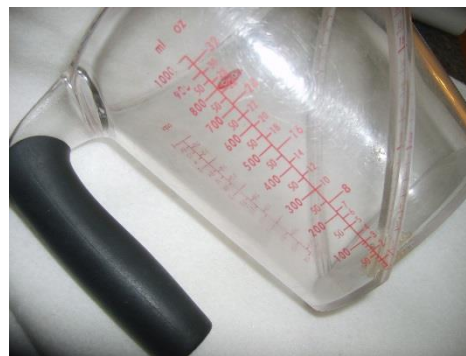
The first two concepts are relevant to daily home and community life, especially, in the kitchen. The latter two emerge more naturally within the context of arithmetic and algebraic experiences—We deal with them in other *OWaM Series*.

Parts of Unit Wholes. We believe that students encounter fractions first and most often in the kitchen. They see measuring cups and spoons, some with gradients on the side. These units evolved long ago and have not incorporated metrics—so the cup and the quart are considered units. We need fractions to name their parts. Snack-time treats like birthday cakes are divided equitably and discussed in terms of fractional parts like halves and fourths.



Such units of measurement can also be described as continuous because each unit is marked off only by imaginary boundaries as opposed to a dozen eggs in which each unit is clearly separate.

So the series anchors fractional ideas about “parts of unit wholes” in activities that develop such images. Activities with fractional parts of geometric shapes like circles and squares take the concept of parts of unit wholes a step out of the kitchen into the more universal and abstract.

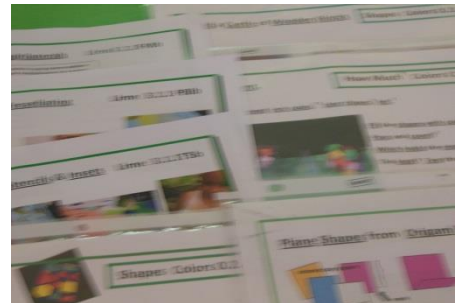


Parts of Collections. As an early image of fractions of sets or collections, we offer eggs in cartons. So part of the whole collection might be six eggs represented as $\frac{1}{2}$. Collections can also be called sets and their fractional parts called subsets.



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